Winding losses calculation for a 10 MW ironless-stator axial flux permanent magnet generator for offshore wind power plant

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Abstract

It is well-known that using expensive Litz wire is an effective solution to reduce eddy current losses in high-speed low-power electrical machines. However, in an offshore wind power application where a direct-drive 10 MW ironless-stator generator and full-scale converter are used, the use of Litz wire becomes also necessary.

Meanwhile, the progress in the manufacture of Roebel winding and Continuously Transposed Conductors (CTC) makes these conventional cheap solutions promising to be employed for a cost-effective generator. This study examines the feasibility of using such technologies instead of Litz wire by studying the winding losses associated with each solution. Completely transposed windings with different number of subconductors per turn were considered. Both resistive and rotational losses sources are explored in this analysis by means of three-dimensional finite element method (3-D FEM) software (i.e. Ansys Maxwell) integrated with an advanced computing server of 48-core and 128 RAM.

In terms of resistive losses, decisive factors are the surface and length of the strand, since paths followed by each subconductor in the interlacement process pose a small influence, especially because of the low frequencies in play and the fact that circulating currents are highly reduced in a complete transposition. On the other hand, rotational losses depend mainly on winding dimensions and orientation. Results show that both proposed technologies are still far from delivering the losses reduction performance provided by Litz wire.

Keywords

Axial flux permanent magnet generator, circulating currents, continuously transposed conductors, eddy losses, Litz wire, Roebel transposition.
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<td>( A )</td>
<td>Magnetic vector potential</td>
</tr>
<tr>
<td>( B )</td>
<td>Magnetic induction vector</td>
</tr>
<tr>
<td>( B_g )</td>
<td>Normal component of magnetic flux density vector in the air gap</td>
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<td>( B_r )</td>
<td>Remanence induction</td>
</tr>
<tr>
<td>( C )</td>
<td>Boundary of the method of weighted residuals</td>
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<tr>
<td>( D )</td>
<td>Electric displacement vector</td>
</tr>
<tr>
<td>( E )</td>
<td>Electric field vector</td>
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<tr>
<td>( E_{cond} )</td>
<td>Conduction electric field vector</td>
</tr>
<tr>
<td>( E_{grad} )</td>
<td>Gradient electric field vector</td>
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<td>( E_{ind} )</td>
<td>Induction electric field vector</td>
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<td>Laplace-Lorentz force</td>
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<td>( F_e )</td>
<td>Electric force vector</td>
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<td>( F_m )</td>
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<td>Geometric mean distance between domain ( d ) of the subconductor ( p ) and domain ( k ) of the subconductor ( q )</td>
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<td>( dV )</td>
<td>Infinitesimal element of a volume</td>
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<tr>
<td>( dp )</td>
<td>Elemental power</td>
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<tr>
<td>( dq )</td>
<td>Infinitesimal amount of free charge</td>
</tr>
<tr>
<td>( dr )</td>
<td>Infinitesimal element along length of conductor</td>
</tr>
<tr>
<td>( ds )</td>
<td>Infinitesimal element of a path</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Frequency</td>
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<td>( i )</td>
<td>Current intensity</td>
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<td>Symbol</td>
<td>Description</td>
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</tr>
<tr>
<td>$m$</td>
<td>Number of phases</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of coils</td>
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<td>$n_s$</td>
<td>Stokes' unit normal</td>
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<td>Number of pole pairs</td>
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<td>Voltage</td>
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<td>Velocity</td>
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<td>$v_f$</td>
<td>Average value of charge velocity vector parallel to the impressed electric field originating the movement</td>
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<td>Eddy losses per unit depth</td>
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<td>Magnetic permeability</td>
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<td>$\rho$</td>
<td>Electric resistivity</td>
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<td>$\rho_f$</td>
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<td>$\sigma$</td>
<td>Electric conductivity</td>
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<tr>
<td>$\psi$</td>
<td>Magnetic flux linkage</td>
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<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
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## List of Abbreviations

<table>
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<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>AFPM</td>
<td>Axial Flux Permanent Magnet</td>
</tr>
<tr>
<td>CTC</td>
<td>Continuously Transposed Conductors</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive Force</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
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<td>MMF</td>
<td>Magnetomotive Force</td>
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<td>Method of Weighted Residuals</td>
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<td>Permanent Magnets</td>
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<td>RFPM</td>
<td>Radial Flux Permanent Magnet</td>
</tr>
<tr>
<td>WWEA</td>
<td>World Wind Energy Association</td>
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Chapter 1

Introduction

In this chapter a brief description of the work is presented. Principles behind wind energy are exposed with special emphasis being given to offshore generation. Basic structure and characteristics of AFPM machines are described and the FEM is introduced. Objectives of the thesis are presented as well as the reasons that supported its development. Finally, the structure of the thesis report is described.
1.1. Wind energy

“With rising costs of energy and growing concerns about the environmental effects of fossil fuel use, researchers have been looking for alternatives as creating power sources. Wind power, a renewable and virtually inexhaustible power source, is a promising means of green energy production. Currently, wind power is not in wide use and accounts for the production of only about 1% of energy used world-wide. However, wind power generation has been considerably increasing since 1999. Wind power is basically converted solar power. As the sun heats the earth, land masses and oceans are heated in varying degrees as they absorb and reflect heat at different rates. This causes portions of the atmosphere to warm differently and as hot air rises, atmospheric pressure causes cooler air to replace it. The resulting movement in the air is wind. The kinetic energy of wind is converted by turbine blades which drive a generator to produce electrical energy. Wind power can be harnessed using wind turbines grouped together on wind farms, located either on land or offshore, for large-scale production. Wind power generation varies in size from small generators, which produce sufficient electrical power for a small farm, to wind farms, which can generate power for thousands of households. The intermittent nature of wind makes reliability and storage of wind energy an important issue. Utilities must maintain sufficient power to meet customer demand plus an additional reserve margin. Although wind is variable and at times does not blow at all, fluctuations in the output from wind farms can be accommodated within normal operating strategies, as the majority of wind is added to power systems as an energy source rather than a capacity source. A spinning reserve enables a plant to meet demand. Amount of energy production is based on the average wind speed at the site of the wind farm and the correlation between output and demand. Capacity of wind farms also depends on geographical dispersion, i.e. the further apart wind plants are located, the greater the chance that some of them will be producing power at any given time. The capacity of the transmission grid to deliver wind energy to customers has been identified as one of the biggest constraints on wind energy use. Often, high altitude windy areas, which are best suited as sites for wind farms, are not located near demand centers, causing the power generated to be transmitted over long distances resulting in losses of power. Moreover, costs are also increased with the necessity of building transmission lines and substations. Environmental concern has been raised over avian fatalities caused by collisions with wind turbines. Although bird deaths caused by wind turbines are low when compared to avian death caused by birds flying into buildings, environmentalists insist on urging that patterns of bird flight and paths of migration should take part into site selection for wind farms. As a result, conservationists, wind power industry officials and federal agencies have joined together to conduct research and reduce the number of birds and bats killed by turbines. Further environmental concerns associated with wind power include visual pollution, noise and erosion. Costs of wind energy have dropped significantly in the last twenty years due to improvements in technology and economies of scale - general increase in wind farm size. With wind power the energy source is free, so the costs associated with wind energy are related to up-front costs of constructing the systems to capture energy in the wind and convert it into electricity. Up-front capital outlays incurred by planning, purchasing of equipment, construction of access roads and foundation and connecting to the grid and installation represent 70% of costs associated with wind power. Maintenance, taxes, insurance and administrative costs make up the remaining 30%. Wind power is the fastest growing power source worldwide on a percentage basis and according to the U.S. Department of Energy global winds could theoretically supply more than fifteen times current world energy demand. Benefits of wind energy include the following” – as seen in [1]:
i. clean and emission free;
ii. inexhaustible and abundant;
iii. domestic resource, reducing the need to rely on foreign sources of power;
iv. growing public support;
v. cost efficient.

“One of the first wind turbines to generate electricity was a traditional wooden windmill converted by Poul la Cour in Denmark over one hundred years ago. In the early part of the twentieth century, there were further experimental machines but serious developments only began with the two oil shocks in the 1970s, when governments around the world reacted by directing Research & Development money to alternatives to fuel sources. The early 1980s saw major developments with the construction of the famous fields of hundreds of small turbines. However, the stabilizing of the oil price in that decade and resulting reductions in the subsidies for wind power meant that purchases from the crucial market all over the world dried up, and many wind turbine companies withdrew from the field or went bankrupt. An exception was Denmark, where government support meant that the knowledge base was not dissolved and the companies there were able to quickly respond when wind energy’s fortunes revived once more in the early 1990s, to the point where they and their partners continue to have a strong position in the market today. It should be pointed out that the foundations of renewable energy’s fortunes are today based on the solid necessity of alleviating climate change and increased energy autonomy rather than the fickle nature of oil prices. According to the World Wind Energy Association (WWEA), there was a total global installed wind power capacity of approximately 238 GW by the end of 2011, being China the most prominent country not only with the largest installed capacity, but also achieving an annual growth of 40%. With the resulting economies of scale, wind energy now competes on price with the traditional generators, such as coal and nuclear, in areas of rich wind resources” – as seen in [2].

1.1.1. Offshore generator technology – state of the art

“Offshore wind farms promise to become an important source of energy in the near future. It is expected that, by the end of this decade, wind parks with a total capacity of thousands of megawatts will be installed in European seas. This will be equivalent to several large traditional coal-fired power stations. Plans are currently advancing for such large-scale wind parks in Swedish, Danish, German, Dutch, Belgian, British and Irish waters. Onshore wind energy has grown enormously over the last decade to the point where it generates more than 10% of all electricity in certain regions (e.g. Denmark, Schleswig-Holstein in Germany and Gotland in Sweden). However, this expansion has not been without problems and the resistance to wind farm developments, experienced in Britain since the mid-1990s, is now present in other countries to a lesser or greater extent. One solution of avoiding land-use disputes and to reduce the noise and visual impacts is to move the developments offshore, which also has a number of other advantages” – as seen in [2]:

i. availability of large continuous areas, suitable for major projects;
ii. higher wind speeds, which generally increase with distance from the shore (Britain is an exception to this as the speed-up factor over hills means that the best wind resources are where the turbines are also most visible);
iii. less turbulence, which allows the turbines to harvest the energy more effectively and reduces the fatigue loads on the turbine;

iv. lower wind-shear (i.e. the boundary layer of slower moving wind close to the surface is thinner), thus allowing the use of shorter towers.

But against this is the very important disadvantage of the additional capital investment necessary, relating to:

i. the more expensive marine foundations;

ii. the more expensive integration with the electrical network and, in some cases, a necessary increase in the capacity of weak coastal grids;

iii. the more expensive installation procedures and restricted access during construction due to weather conditions;

iv. limited access for Observations & Measurements during operation which results in an additional penalty of reduced turbine availability and hence reduced output.

“However, the cost of wind turbines is falling and is expected to continue doing so over the coming decade. Once sufficient experience has been gained in building offshore projects, the offshore construction industry is likely to find similar cost-savings. At locations with good wind speeds, onshore wind energy has become a cost-competitive resource at a stable price compared to conventional power generation, especially when environmental benefits are accounted for. Hence, it would seem likely that offshore wind energy will also become competitive in time. Other developments that are likely to support this trend are the design of turbines optimized for the offshore environment, of greater sizes (i.e. up to 10 MW and over 125 meters of rotor diameter) and with greater reliability built-in. Over the last ten and five years, the average offshore generator are 2.63 MW and 2.89 MW, respectively. At the moment, the average generator power in the wind turbines available on the market is 3.8 MW, which implies the industry is experiencing a considerable power rating increase [3]. With full-scale serial manufacturing generally a couple of years behind, the middle of this decade should allow a developer to choose between several competing machines. The wind turbine manufacturing industry has been following its own exponential growth curves over the last decade of decreasing costs by 20%, increasing annual installed capacity by 50% and doubling the size of the largest commercially-available turbine every three or so years. The average power rating of installed offshore wind power generator increases with a rate of 0.25 MW per year, and will reach 4 MW after 2014 as long as the market available designs become proven as expected [3]. The total wind power resources available offshore are vast and will certainly be able to supply a significant proportion of electricity needs in an economic manner” – as seen in [2].

“The wind turbines being used in current offshore projects tend to be machines designed for land-use but with modifications, such as a larger generator, a higher instrumentation specification and component redundancy, particularly of electrical systems. If the market expands as expected, machines designed for optimized performance offshore will be developed and utilized but it is not certain how they will look. On one hand, the requirements from an offshore machine differ from those on land; on the other hand, the requirement for high reliability would suggest the use of well-proven turbines. Modifications may include” – as seen in [2]:

i. larger machines, up to 5 MW or 10 MW;
ii. faster rotational speeds than on land, where noise restrictions generally mean that the turbine operates slightly below optimum speed;

iii. larger generators for a specific rotor size, to enable the additionally available energy to be efficiently harvested;

iv. high voltage generation, also possible in DC instead of AC.

“In the longer term, downwind machines with flexible blades or multiple rotors might become an option, but engineering effort will be needed to achieve the claimed theoretical potential. The economics of offshore wind energy encourage the development of very large wind turbines in order to justify the additional investment necessary for the more expensive support structures, grid connection and installation procedures. The economics will become particularly important in the deeper waters of interest to German developers and, in the longer term, elsewhere as well, when the most suitable near-shore sites have already been developed or visual-impact becomes an important issue” – as seen in [2].

“It would seem that the current optimism about offshore wind energy has a firm basis in currently available technology, in likely reductions in cost and, of equal importance, in the general widespread public and political support and the generally low impact on the environment. The experience of the first prototype offshore wind farms has proven the technical viability and the large-scale developments currently being undertaken will bring in much practical experience on issues such as construction methods, installation procedures, access and Observations & Measurements philosophies, which will result in a more economic generation cost of electricity” – as seen in [2].

Brushless permanent magnet (PM) electrical machines are the primary generators for distributed generation systems. They are compact, high efficient and reliable self-excited generators. The distributed generation is any electric power production technology that is integrated within a distribution system. Distributed generation technologies are categorized as renewable and nonrenewable. Renewable technologies include solar, photovoltaic, thermal, wind, geothermal and ocean as sources of energy. Nonrenewable technologies include internal combustion engines, combined cycles, combustion turbines, micro turbines and fuel cells. Axial flux permanent magnet (AFPM) brushless generators can be used both as high speed and low speed generators. Their advantages are high power density, modular construction, high efficiency and easy integration with other mechanical components like turbine rotors or flywheels. The output power is usually rectified and then inverted to match the utility grid frequency or only rectified. A low speed AFPM generator is usually driven by a wind turbine. With wind power rapidly becoming one of the most desirable alternative energy sources world-wide, AFPM generators offer the ultimate low cost solution as compared with, for instance, solar panels [4].

### 1.2. Axial flux permanent magnet machines

The AFPM machine, also called the disc-type machine, is an attractive alternative to the cylindrical radial flux permanent magnet (RFPM) machine due to its pancake shape, compact construction and high power density. AFPM motors are particularly suitable for electrical vehicles, pumps, fans, valve control, centrifuges, machine tools, robots and industrial equipment. AFPM machines can also operate as small to medium power generators. Since a large number of poles can be accommodated, these machines are ideal for low speed applications, as for
example, electromechanical traction drives, hoists or wind generators. The unique disc-type profile of the rotor and stator of AFPM machines makes it possible to generate diverse and interchangeable designs. AFPM machines can be designed as single air gap or multiple air gaps machines, with slotted, slotless or even totally ironless armature. Low power AFPM machines are frequently designed as machines with slotless windings and surface permanent magnets [4].

The history of electrical machines reveals that the earliest machines were axial flux machines. M. Faraday’s first primitive working prototype of an axial flux machine ever recorded, in 1831; anonymous inventor with initials P.M., in 1832; W. Ritchie, in 1833; and B. Jacobi, in 1834. However, shortly after T. Davenport claimed the first patent for a radial flux machine, in 1837, conventional radial flux machines have been widely accepted as the mainstream configuration for electrical machines [4]. Several reasons arise for shelving the axial flux machine, from the strong axial magnetic attraction force between stator and rotor, to the high costs involved in manufacturing the laminated stator cores, whose fabrication cut is regarded as a difficult task as the assembling of the machine itself and the requirement to keep a uniform air gap. Although the first PM excitation system was applied to electrical machines as early as the 1830s, the poor quality of hard magnetic materials soon discouraged their use. The invention of Alnico in 1931, barium ferrite in the 1950s and especially the rare-earth neodymium-iron-boron (NdFeB) material, in 1983, have made a comeback of the PM excitation system possible. It is generally believed that the availability of high energy PM materials is the main driving force for exploitation of novel PM machine topologies and has thus revived the use of AFPM machines. Prices of rare-earth PMs have been following a descending curve in the last decade of the twentieth century. With the availability of more affordable PM materials, AFPM machines may play a more important role in the near future [4].

From a construction point of view, brushless AFPM machines can be designed as single-sided or double-sided, with or without armature slots, with or without armature core, with internal or external PM rotors, with surface mounted or interior PMs and as single stage or multi-stage machines. In the case of double-sided configurations, either the external stator or external rotor arrangement can be adopted. The first choice has the advantage of using fewer PMs at the expense of poor winding utilization while the second one is considered as a particularly advantageous machine topology. The diverse topologies of AFPM brushless machines may be classified as follows [4]:

i. single-sided AFPM machines:
   a. with slotted stator;
   b. with slotless stator;
   c. with salient-pole stator.

ii. double-sided AFPM machines:
   a. with internal stator:
      1. with slotted stator;
      2. with slotless stator:
         i. with iron core stator;
         ii. with coreless stator;
         iii. without both rotor and stator cores.
3. with salient pole stator.

b. with internal rotor:
   1. with slotted stator;
   2. with slotless stator;
   3. with salient pole stator.

iii. multidisc AFPM machines.

The machine studied in this research is double-sided with internal coreless stator.

The air gap of the slotted armature AFPM machine is relatively small. The mean magnetic flux density in the air gap decreases under each slot opening due to increase in the reluctance. For AFPM machines with slotless windings the air gap is much larger and compared to a conventional slotted winding, the slotless armature winding has advantages such as simple stator assembly, elimination of the cogging torque (i.e. the torque due to the interaction between the permanent magnets of the rotor and the stator slots) and reduction of rotor surface losses, magnetic saturation and acoustic noise. The disadvantages include the use of more PM material, lower winding inductances sometimes causing problems for inverter-fed motors and significant eddy current losses in slotless conductors. Depending on the application and operating environment, slotless stators may have ferromagnetic cores or be completely coreless. Coreless stator configurations eliminate any ferromagnetic material from the armature system, thus making the associated eddy current and hysteresis core losses nonexistent. This type of configuration also eliminates axial magnetic attraction forces between the stator and rotor at zero-current state [4].

In pace with the application of new materials, innovation in manufacturing technology and improvements in cooling techniques, further increase in the power density (output power per mass or volume) of the electrical machine has been made possible [4]. There is an inherent limit to this increase for conventional RFPM machines because of:

i. the bottle-neck feature for the flux path at the root of the rotor tooth in the case of induction and DC commutator machines or brushless machines with external rotors, as illustrated in Figure 1-1;
ii. much of the rotor core around the shaft (rotor yoke) is hardly utilized as a magnetic circuit;
iii. heat from the stator winding is transferred to the stator core and then to the frame (i.e. there is poor heat removal through the stator air gap, rotor and shaft without forced cooling arrangements).

![Figure 1-1 - Topologies of RFPM machine (a) and AFPM machine (b) - Source: [4]](image-url)
These limitations are inherently bound with radial flux structures and cannot be removed easily unless a new topology is adopted. The AFPM machine, recognized as having a higher power density than the RFPM machine, is more compact than its radial flux counterpart. Moreover, since the inner diameter of the core of an AFPM machine is usually much greater than the shaft diameter, better ventilation and cooling can be expected [4]. In general, the special properties of AFPM machines, which are considered advantageous over RFPM machines in certain applications, can be summarized as follows:

i. AFPM machines have much larger diameter to length ratio than RFPM machines;
ii. AFPM machines have a planar and somewhat adjustable air gap;
iii. capability of being designed to possess a higher power density with some saving in core material;
iv. the topology of an AFPM machine is ideal to design a modular machine in which the number of the same modules is adjusted to power or torque requirements;
v. the larger the outer diameter of the core, the higher the number of poles that can be accommodated, making the AFPM machines a suitable choice for high frequency or low speed operations.

The power range of AFPM disc-type brushless machines is now from a fraction of a Watt to sub-MW. As the output power of the AFPM machine increases, the contact surface between the rotor and shaft becomes smaller in comparison with the rated power. It is more difficult to design a high mechanical integrity rotor-shaft joint in the higher range of the output power. A common solution to the improvement of the mechanical integrity of the rotor-shaft joint is to design a multidisc machine. Since the scaling of the torque capability of the AFPM machine as the cube of the diameter while the torque of a RFPM machines scale as the square of the diameter times the length, the benefits associated with axial flux geometries may be lost as the power level or the geometric ratio of the length to diameter of the motor is increased. The transition occurs near the point where the radius equals twice the length of RFPM machine. This may be a limiting design consideration for the power rating of a single-stage disc machine as the power level can always be increased by simply stacking of disc machines on the same shaft and in the same enclosure [4].

1.2.1. Materials and fabrication

Magnetic circuits of rotors consist of PMs and mild steel backing rings or discs. Since the air gap is somewhat larger than that in similar RFPM counterparts, high energy density PMs should be used. Normally, surface magnets are glued to smooth backing rings or rings with cavities of the same shape as magnets without any additional mechanical protection against normal attractive forces. Epoxy, acrylic or silicon based adhesives are used for gluing between magnets and backing rings or between magnets. There were attempts to develop interior PM rotor for AFPM machines. Rotor poles can only be fabricated by using soft magnetic powders. The main advantage of this configuration is the improved flux weakening performance. However, the complexity and high cost of the rotor structure discourage further commercializing development.

A PM can produce magnetic flux in an air gap with no exciting winding and no dissipation of electric power. As any other ferromagnetic material, a PM can be described by its hysteresis loop. PMs are also called hard magnetic material, which means ferromagnetic materials with a wide hysteresis loop, as will be explained with more detail further on this report. There are three classes of PMs currently used for electric machines:
i. Alnicos (Al, Ni, Co, Fe);
ii. Ceramics (ferrites), e.g. barium ferrite and strontium ferrite;
iii. Rare-earth materials, e.g. samarium-cobalt and neodymium-iron-boron.

Alnico magnets dominated the PM motor market in the range from a few watts to 150 kW between the mid-1940s and the late 1960s. The main advantages of Alnico are its high magnetic remanent flux density and low temperature coefficients. The temperature coefficient of remanence induction is -0.02% per degree Celsius and maximum service temperature is 520 degrees Celsius. Unfortunately, the coercive force is very low and the demagnetization curve is extremely non-linear. Therefore, it is very easy not only to magnetize but also to demagnetize Alnico. Alnico has been used in PM DC commutator motors of the disc type with relatively large air gaps. This results in a negligible armature reaction magnetic flux acting on the PMs. Sometimes, Alnico PMs are protected from the armature flux, and consequently from demagnetization, using additional mild steel pole shoes [4].

Barium and strontium ferrites produced by powder metallurgy were invented in the 1950s. Ferrite magnets are available in isotropic and anisotropic grades. A ferrite has a higher coercive force than Alnico, but at the same time has a lower remanent magnetic flux density. Temperature coefficients are relatively high, i.e. the coefficient of remanence induction is -0.20% per degree Celsius and the coefficient of coercive field is -0.27 to -0.4% per degree Celsius. The maximum service temperature is 450 degrees Celsius. The main advantages of ferrites are their low cost and very high electric resistance, which means practically no eddy-current losses in the PM volume [4].

The first generation of rare-earth permanent magnets has been commercially produced since the early 1970s. It had the advantage of a high remanent flux density, high coercive force, high energy product, a linear demagnetization curve and a low temperature coefficient. The temperature coefficient of remanence induction is -0.02 to -0.045% per degree Celsius and the temperature coefficient of coercive field is -0.14 to -0.40% per degree Celsius. Maximum service temperature is 300 to 350 degrees Celsius. It is suitable for motors with low volumes and motors operating at increased temperatures, e.g. brushless generators for micro turbines. Elements that make up this first generation (i.e. Sm and Co) are relatively expensive due to their supply restrictions. With the discovery in the recent years of a second generation of rare-earth magnets on the basis of inexpensive neodymium (Nd), remarkable progress with regard to lowering raw material costs has been achieved. The new generation of rare-earth PMs based on inexpensive Nd was announced in 1983. The Nd is a much more abundant rare-earth element than Sm. NdFeB magnets, which are now produced in increasing quantities have better magnetic properties than those of SmCo, but unfortunately only at room temperature. The demagnetization curves, especially the coercive force, are strongly temperature dependent. The temperature coefficient of remanence induction is -0.09 to -0.15% per degree Celsius and the temperature coefficient of coercive field is -0.40 to -0.80% per degree Celsius. The maximum service temperature is 250 degrees Celsius. The NdFeB is also susceptible to corrosion. NdFeB magnets have great potential for considerably improving the performance-to-cost ratio for many applications. For this reason they will have a major impact on the development and application of PM machines in the future [4].
Magnetic circuits of rotors of AFPM brushless machines provide the excitation flux and are designed as PMs glued to a ferromagnetic ring or disc which serves as a backing magnetic circuit (yoke), or PMs arranged into Halbach array without any ferromagnetic core. Shapes of PMs are usually trapezoidal, circular or semicircular, as illustrated in Figure 1-2. The shape of PMs affects the distribution of the air gap magnetic field and contents of higher space harmonics. The output voltage quality (harmonic content of the generated EMF) of AFPM generators depends on the PM geometry (circular, semicircular, trapezoidal) and distance between adjacent magnets. Since the magnetic flux in the rotor magnetic circuit is stationary, mild steel (carbon steel) backing rings can be used [4].

The key concept of Halbach array is that the magnetization vector of PMs should rotate as a function of distance along the array, as exposed in Figure 1-3, providing the following advantages:

i. the fundamental field is stronger by a factor of 1.4 than in a conventional PM array, and thus the power efficiency of the machine is doubled;

ii. the array of PMs does not require any backing steel magnetic circuit and PMs can be bonded directly to a non-ferromagnetic supporting structure (e.g. aluminum and plastics);

iii. the magnetic field is more sinusoidal than that of a conventional PM array;

iv. Halbach array has very low back-side fields.

Figure 1-3 - A Halbach array, showing the orientation of each piece's magnetic field. This array would give a strong field underneath, while the field above would cancel - Source: http://en.wikipedia.org/wiki/File:Halbach_array.png.
Armature windings of electric motors are made of solid copper conductor wires with round or rectangular cross sections, whose conductivity is temperature dependent. The maximum temperature rise for the windings of electrical machines is determined by the temperature limits of insulating materials. A polyester-imide and polyamide-imide coat can provide an operating temperature of 200 degrees Celsius. The highest operating temperatures (over 600 degrees Celsius) can be achieved using nickel clad copper or palladium-silver conductor wires and ceramic insulation [4].

Stator coreless windings of AFPM machines are fabricated as uniformly distributed coils on a disc-type cylindrical supporting structure (hub) made of nonmagnetic and nonconductive material. There are two types of windings:

i. winding comprised of multi-turn coils wound with turns of insulated conductor of round or rectangular cross section;

ii. printed winding also called film coil winding.

Coils are connected in groups to form the phase windings typically connected in star or delta. Coils or groups of coils of the same phase can be connected in parallel to form parallel paths. To assemble the winding of the same coils and obtain high density packing, coils should be formed with offsetting bends, as shown in Figure 1-4. The space between two sides of the same coil is filled with coil sides from each of the adjacent coils [4].

Figure 1-4 - Disc-type coreless winding assembled of coils of the same shape: (a) single coil; (b) three adjacent coils - Source: [4]. 1 – coil side. 2 - inner offsetting bend . 3 - outer offsetting bend.

Coils can be placed in a slotted structure of the mold. With all the coils in position, the winding (often with a supporting structure or hub) is molded into a mixture of epoxy resin and hardener and then cured in a heated oven. Because of the difficulty of releasing the cured stator from the slotted structure of the mold, each spacing block that forms a guide slot consists of several removable pins of different size. For very small AFPM machines and micro machines printed circuit coreless windings allow for automation of production. Printed circuit windings for AFPM brushless machines fabricated in a similar way as printed circuit boards have not been commercialized due to poor performance. A better performance has been achieved using film coil windings made through the same process as flexible printed circuits. The coil pattern is formed by etching two copper films that are then attached to both sides of a board made of insulating materials. Compact coil patterns are made possible by connecting both sides of coil patterns through holes [4].
1.2.2. Double-sided machines with a coreless stator

In this section the basic principles of double-sided AFPM machine with a coreless stator are superficially discussed. Detailed explanation about the electromagnetic principles concerning stator winding losses is given in the upcoming sections.

AFPM machines with coreless stators have the stator winding wound on a non-magnetic and non-conductive supporting structure or mold. Therefore, the stator core losses (i.e. hysteresis and eddy current losses) do not exist. Moreover, the losses in PMs and rotor solid steel disc are negligible. However, depending on the frequency, significant eddy current losses in the stator winding conductors may occur. Since there is non-magnetic material enforcing the magnetic field in the stator, a much larger volume of PMs in comparison with laminated stator core AFPM machine is required in order to maintain a reasonable level of flux density in the air gap. As illustrated in Figure 1-5, the stator winding is placed in the air gap magnetic field generated by the PMs mounted on two opposing rotor discs [4].

![Figure 1-5 - Basic topology of a double-sided AFPM machine with a coreless stator - Source: [4]. 1 – stator winding. 2 – rotor. 3 – PM. 4 – frame. 5 – bearing. 6 – shaft.](image)

Coreless configurations eliminate any ferromagnetic material from the armature, thus eliminating the associated eddy current and hysteresis core losses. Because of the absence of core losses, a coreless stator AFPM machine can operate at higher efficiency than conventional machines. On the other hand, owing to the increased nonmagnetic air gap, such a machine uses more PM material than an equivalent machine with a ferromagnetic stator core [4].

For the ease of construction, the stator winding normally consists of a number of single layer trapezoidal shaped coils. The assembly of the stator is made possible by bending the ends of the coils by a certain angle, so that the active conductors lie evenly in the same plane and the end windings nest closely together. The windings are held together in position by using a composite material of epoxy resin and hardener [4].
In a machine with \( p \) pole pairs and \( m \) phases, the total number of coils is \( n_c = mp \). Each coil has two sides in the active region of the machine. Therefore, each segment of the machine containing one pole pair has \( 2m \) coil sides. Figure 1-6 shows the coreless stator winding of a three-phase, eight-pole AFPM machine, where winding arrangement per phase and coil side numbering can be seen. Another coil profile that has been used in coreless stator AFPM machines is the rhomboidal coil. It has shorter end connections than the trapezoidal coils. The inclined arrangement of the coil’s active sides makes it possible to place water cooling ducts inside the stator. The main drawback of rhomboidal coils is the reduction of the torque [4].

![Figure 1-6 - Coreless winding of a three-phase, eight-pole AFPM machine with twin external rotor - Source: [4].](image)

### 1.3. Overview of the finite element method

The field of Electrical Engineering may be subdivided into three major areas. Firstly, Theoretical Electromagnetism, that deals with fundamental laws and principles of electromagnetism studied for their intrinsic scientific value. Secondly, Applied Electromagnetism, that transfers the theoretical knowledge to scientific and engineering applications, as it regards the construction of mathematical models of physical phenomena. Finally, Computational Electromagnetism, that solves specific problems by simulation through numerical methods implemented on digital computers [5].

Analysis and design of electrical equipment is a difficult task not only due to the complex geometry and mixed set of materials (some of which have nonlinear characteristics) involved, but also because mixed phenomena are present (i.e. electromagnetic field, thermal and mechanical aspects). The first step of analysis is the definition of the universe of analysis, which basically means deciding what aspects should be considered and what may be neglected. This step is called Modeling and is concern of Theoretical Electromagnetism. The second step of analysis is the selection of a numerical method to solve the problem. This process is named Discretization and is concern of Computational Electromagnetism. Finite element method (FEM) is the most used method to linear and nonlinear problems without restrictions on the geometry. Finally, the third step of analysis is computation of additional results and analysis of the solution to evaluate the results and ascertain if the analysis must be repeated with an higher level of discretization. This aspect is the concern of Applied Electromagnetism [5].
In essence, the finite element is a mathematical method for solving ordinary and partial differential equations. Since it is a numerical method, it has the ability to solve complex problems that can be represented in differential equation form. As these types of equations occur naturally in virtually all fields of the physical sciences, the applications of the finite element method are limitless as regards to the solution of practical design problems. Due to the high cost of computing power of past years, FEM has a history of being used to solve complex and cost critical problems. Classical methods alone usually cannot provide adequate information to determine, for instance, the safe working limits of a major civil engineering construction. If a tall building, a large suspension bridge or a nuclear reactor failed catastrophically, the economy and social costs would be unacceptably high. In recent years, FEM has been used almost universally to solve structural engineering problems. Nowadays, even the most simple of products rely on FEM for design evaluation, because contemporary design problems usually cannot be solved as accurately and cheaply using any other method currently available. Physical testing was the norm in years gone by, but now it is simply too expensive. Many kinds of electromagnetic phenomenon can be modeled, from the propagation of microwaves to the torque in an electromagnetic motor. Analysis of electromagnetic field passing through and around a structure provides insight into the response and hence a means for regulating these fields to attain specific responses [5].

1.4. Objective of the thesis

The main objective of the thesis work presented in this report is to compare the losses performance of CTC and Roebel winding when working in the coreless stator of a double-sided AFPM machine. The idea is to examine the feasibility of using such technologies instead of the more expensive Litz wire solution. This study focuses on comparing the two mentioned windings in terms of rotational and Joule losses, considering all aspects inherent to a transposed coil, including circulating currents between parallel subconductors. Dimension independent scripts to build each of the Maxwell models necessary to calculate losses parameters were developed using Matlab.

1.5. Outline of the thesis report

The structure of this thesis follows the work evolution from basic analytical study to the complex computational models. Thus, in this first chapter, a brief description of the work is presented. Principles behind wind energy are exposed with special emphasis being given to offshore generation. Basic structure and characteristics of AFPM machines are described and the FEM is introduced. The second chapter presents a brief overview on electromagnetic foundations, moving from the most general axioms to the specificities of a double-sided coreless stator AFPM machine, moving from there to the phenomena linked with stator winding losses. The third chapter describes in detail the computational methods used to calculate stator winding losses. Firstly, the three-dimensional model of the segment used to study the machine is presented, with special emphasis being given to the architecture of the two transposed winding technologies under analysis. Results for resistive and rotational losses are presented and the critical aspects behind the methods’ algorithms discussed. Finally, the fourth chapter is aimed at detailing overall conclusions of the thesis as well as presenting future work perspectives.
Chapter 2

The problem

To understand the principles behind any electrical machine, it is crucial to have a solid background on electromagnetic foundations. In this chapter a brief overview on that field is provided, moving from the most general axioms to the specificities of a double-sided coreless stator AFPM machine, moving from there to the phenomena linked with stator winding losses. Finally, a general analytical example on the FEM is given and the computational approach chosen to solve the proposed task is described.
2.1. Electromagnetic foundations

To understand the concepts under analysis in this report, basic knowledge of magnetic and electric field theory is necessary. The general concepts here introduced refer to the physics behind electromagnetic rotating machinery, by means of which the bulk of energy conversion takes place. They are constantly utilized throughout the rest of this report. The geometric algebra formulations needed to understand the concepts from now on described can be found in Appendix A.

2.1.1. Basic field and force vectors

From the set of Maxwell’s equations, the following is of the most importance in this analysis.

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  \hspace{2cm} (2.1)

\( \vec{E} \) is the electric field vector and \( \vec{B} \) is the magnetic induction vector. It basically means that if \( \vec{B} \) happens to be a time-varying field then it will give rise to the presence of an electric field \( \vec{E} \) (induction phenomena), as illustrated in Figure 2-1. This equation, along with the others from the set, is axiomatic; it cannot be mathematically deduced from any source. It is simply a result of experimental research accumulated until the end of the nineteenth century [6].

![Figure 2-1](image)

Figure 2-1 - Time-varying magnetic fields give rise to electric fields - Source: [6].

The fact that lines of magnetic induction vector are closed, expressed by Maxwell’s equation (2.2), is also taken into account in some deductions along this report.

\[ \nabla \cdot \vec{B} = 0 \]  \hspace{2cm} (2.2)

Another important relation to understand the phenomenology of the machine under analysis is Maxwell’s equation (2.3) that states that electric currents produce magnetic fields. The equation also implies electric induction phenomena. It was in fact the introduction of the term displacement current density \( \frac{\partial \vec{D}}{\partial t} \) that granted Maxwell the honor of having his name associated with the famous set.
\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \] (2.3)

\( \vec{H} \) is the magnetic field vector that relates with the magnetic induction vector by \( \vec{B} = \mu \vec{H} \) where \( \mu \) is the magnetic permeability of the medium. \( \vec{D} \) is the electric displacement vector related to the electric field vector by \( \vec{D} = \varepsilon \vec{E} \) where \( \varepsilon \) is the electric permittivity of the medium.

Electromagnetic fields cannot be created from a void. Their existence requires the presence of sources, namely charges, either at rest or moving in space. Moving charges (i.e. when currents are present) are characterized by a current density vector \( \vec{J} \), that relates with the electric field vector by the electric conductivity \( \sigma \), as shown in equation (2.4).

\[ \vec{J} = \sigma \vec{E} \] (2.4)

A possible physical interpretation for the current density vector is expressed in equation (2.5).

\[ \vec{J} = \rho_f \vec{v}_f \] (2.5)

\( \rho_f \) represents the free charge per unit volume, while \( \vec{v}_f \) denotes the average value of the charge velocity parallel to the impressed electric field originating the movement.

Placing a static \( q \) charged particle in a region where an electric field \( \vec{E} \) is present will cause a force \( \vec{F}_e \) to be exerted on the charge, as shown in equation (2.6).

\[ \vec{F}_e = q \vec{E} \] (2.6)

If the same charged particle is moving with velocity \( \vec{v} \) in a region where a magnetic induction field \( \vec{B} \) exists, a force \( \vec{F}_m \) arises according to equation (2.7).

\[ \vec{F}_m = q (\vec{v} \times \vec{B}) \] (2.7)

Naturally, if a moving charged particle is submitted to an electromagnetic field, the two forces will add up, causing the so-called Laplace-Lorentz force expressed below.

\[ \vec{F} = q (\vec{E} + \vec{v} \times \vec{B}) \] (2.8)
Since the dimensions of AFPM machines are functions of the radius, the electromagnetic torque is produced over a continuum of radii [4]. The tangential force acting on the disc can be calculated on the basis of Ampere’s equation (2.7), given for one charge $q$ moving with velocity $v$. In a current carrying conductor, there are several charges moving, each of them contributing to the magnetic force. Taking into account equation (2.5), the magnetic force surface density can be written as $\overrightarrow{f_m} = \overrightarrow{J} \times \overrightarrow{B}$, which in a conductor with constant surface, like a stator’s coil, will result in a magnetic force within a longitudinal element $dr$ of the conductor given by equation (2.9).

$$dF_m = I_a \left( \overrightarrow{dr} \times \overrightarrow{B} \right) \quad (2.9)$$

Since the whole machine has $2mp$ coil sides with $N$ turns, each one contributing $dF_m$ to the tangential force acting on the disc, the latter can be expressed by equation (2.10), where $\overrightarrow{B}_g$ is the vector of the normal component (i.e. perpendicular to the disc surface) of the magnetic flux density in the air gap. An AFPM disc-type machine provides $\overrightarrow{B}_g$ practically independent of the radius [4]. The total force results simply from the line integral along the winding length.

$$dF_{m_{\text{Total}}}^{\text{Total}} = 2mpNI_a \left( \overrightarrow{dr} \times \overrightarrow{B}_g \right) \quad (2.10)$$

The electromagnetic torque developed by a coreless AFPM brushless machine is produced by the interaction between the open space current-carrying conductor and PMs (Lorentz force theorem). Recalling that the torque is the tendency of a force to rotate an object about an axis, given by the external product between the radius and the tangential force, the electromagnetic torque of the machine $\overrightarrow{T}$ is naturally obtained in equation (2.11), where $r_{\text{in}}$ and $r_{\text{out}}$ are the inner and outer radius of the machine.

$$\overrightarrow{T} = \int_{r_{\text{in}}}^{r_{\text{out}}} \overrightarrow{r}dF_{m_{\text{Total}}} \overrightarrow{dr}$$

$$\overrightarrow{T} = \int_{r_{\text{in}}}^{r_{\text{out}}} \overrightarrow{r}dF_{m_{\text{Total}}} \overrightarrow{dr} \quad (2.11)$$

### 2.1.2. Joule losses

Figure 2-2 - Definitions of electric voltage (a) and current intensity (b) - Source: [6].
Voltage $u$ between two points $a$ and $b$ is a scalar quantity defined as the line integral of the electric field vector between those points, as shown in equation (2.12), where vector $d\vec{s}$ is an infinitesimal element of the path length between $a$ and $b$.

$$u = \int_{ab} \vec{E} \cdot d\vec{s}$$  \hspace{1cm} (2.12)

The current intensity $i$ flowing in a conductor is a scalar quantity defined as a surface integral corresponding to the flux of the current density vector $\vec{J}$ across a conductor section $S$, as shown in equation (2.13), where $dS$ is an infinitesimal element of area belonging to $S$, and $\vec{n}$ is a unit normal.

$$i = \int_{S} \vec{J} \cdot \vec{n} \cdot dS$$  \hspace{1cm} (2.13)

When a voltage $u$ is applied between a piece of conducting material with two accessible terminals, a current of intensity $i$ will flow along the device. If the conducting material behaves as a linear medium (e.g. $\vec{J} = \sigma \vec{E}$), then $i$ and $u$ are proportional, being the proportionality constant the resistance $R$, as expressed in equation (2.14) also known as Ohm’s law.

$$R = \frac{u}{i} = \frac{\int_{ab} \vec{E} \cdot d\vec{s}}{\int_{S} \sigma \vec{E} \cdot \vec{n} \cdot dS}$$  \hspace{1cm} (2.14)

Conductors increase their temperature when submitted to currents. This so-called Joule effect is associated with electron random collisions inside the atomic lattice of the medium. The energy dissipated by this process may vary from point to point inside the conductor. Considering Figure 2-3, where an infinitesimal volume $dV$ of the conductor contains an infinitesimal amount of free charge $dq$. Under the influence of the impressed $\vec{E}$ field, the free charge $dq$ drifts with a velocity $\vec{v}$ driven by an elemental electric force $d\vec{F}_e = dq \vec{E}$, as already seen in equation (2.6).

![Figure 2-3 - Vectors involved in the analysis of Joule losses in a conductor - Source: [6].](image-url)
The activity of this force produces an elemental power $dp$ which is dissipated in $dV$ as shown in set of equations (2.15).

$$dq = \rho_j dV$$
$$dp = \vec{v} \cdot dF_v = \vec{v} \cdot (dq \vec{E}) = \rho_j \vec{v} \cdot \vec{E} dV = p_j dV \quad (2.15)$$

Taking into account equation (2.5), integration of the above result over the conductor’s whole volume $V$ yields the total power losses expressed below.

$$P_j = \int_V p_j dV = \int_V \vec{J} \cdot \vec{E} dV = \int_V \sigma E^2 dV \quad (2.16)$$

The local power losses density $p_j$ is proportional to the squared norm of electric field vector, meaning that hot spots in a conductor are regions where $\vec{E}$ has attained increased values. Considering a parallel-plate structure conductor with a cross section surface $S$ and length $\delta$, then $J = \frac{i}{S}$ and $E = \frac{u}{\delta}$. Therefore, equation (2.17) is reached for Joule losses.

$$P_j = \int_V \frac{ui}{S\delta} dV = \int_V \frac{ui}{S\delta} dV = \int_V \frac{ui}{S\delta} S\delta = Ri^2 \quad (2.17)$$

### 2.1.3. Ampère’s law

Disregarding the displacement current density term on Maxwell’s equation (2.3) and applying Stokes’ theorem to the curl of magnetic field vector, equation (2.18) is found, also called Ampère’s law.

$$\int_{S_i} \vec{\nabla} \times \vec{H} \cdot \vec{n}_s dS = \oint_{\vec{s}} \vec{H} d\vec{s} = \int_{S_i} \vec{J} \cdot \vec{n}_s dS \quad (2.18)$$

$s$ is a closed oriented path, $S_i$ is an open surface having the path $s$ as its bounding edge, and $\vec{n}_s$ is the Stokes’ unit normal. The cyclic integral on the left-hand side of (2.18) is the so-called magnetomotive force (MMF). The result is illustrated in Figure 2-4.
2.1.4. Induction law

The inductance concept is better introduced by considering a conductor loop in air, as illustrated in Figure 2-5. Recalling Ampère’s law, when a current $i$ is made to flow in the conductor loop, a magnetic induction field $\vec{B}$ with closed lines embracing the conductor will be produced.

![Figure 2-5 - Magnetic flux linked with a current-carrying conductor loop - Source: [6].](image)

Defining a closed integration path $\vec{s}$ coinciding with the conductor loop and oriented according to the reference direction assigned to $i$. Further, considering $S_j$ as an open surface having the path $\vec{s}$ as its bounding edge. The unit normal $\vec{n}_s$ to $S_j$ is defined according to the path orientation using Stokes’ rule. When $\vec{B}$ is integrated across $S_j$, as shown in equation (2.19), a quantity called the magnetic flux linkage $\psi$ is obtained.

$$\psi = \int_{S_j} \vec{B} \cdot \vec{n}_s dS$$ (2.19)

When in presence of linear medium (i.e. $\vec{B} = \mu \vec{H}$), by Ampère’s law, the intensity of $\vec{B}$ is proportional to $i$. Likewise, the same will happen with $\psi$ and the proportionality constant between $\psi$ and $i$ is the inductance $L$. This parameter depends only on the permeability of the material media and on the geometry of the inductor. For nonlinear media the inductance concept is meaningless [6].
The analysis of magnetic coupling phenomena is of crucial importance for the problem under study on this report. Considering the system of two conductors illustrated in Figure 2-6, and assuming linear behavior (i.e. saturation and hysteresis are neglected), the magnetic fluxes linked with the conductors 1 and 2 can be expressed as linear combinations of their own currents, as shown in set of equations (2.20).

\[
\psi_1 = \int_{S_1} \overrightarrow{B}(i_1, i_2) \cdot \overrightarrow{n}_1 dS = L_{11}i_1 + L_{12}i_2 \\
\psi_2 = \int_{S_2} \overrightarrow{B}(i_1, i_2) \cdot \overrightarrow{n}_2 dS = L_{21}i_1 + L_{22}i_2
\]

In the general case of n-coupled circuits, a square n-sized real matrix is obtained. This is called the inductance matrix. Its entries can be determined experimentally by measuring fluxes and currents, they can be found numerically using dedicated computer programs, as in the work described in this report, and in some cases, when very simple geometries are considered, they can also be determined analytically [6].

The cornerstone of magnetic induction phenomena is the Maxwell–Faraday induction law [6], described in equation (2.21) and obtained by applying Stokes’ theorem to Maxwell’s equation (2.1). The left-hand side of (2.21) is traditionally known by the name of electromotive force (EMF).

\[
\oint \overrightarrow{E} \cdot d\overrightarrow{s} = -\frac{d}{dt} \int_{S} \overrightarrow{B} \cdot \overrightarrow{n} dS = -\frac{d\psi}{dt}
\]

### 2.1.5. Eddy currents

When a current-carrying conductor is exposed to a time-varying magnetic field, as illustrated in Figure 2-7, electric field closed loops arise from which other currents will result. These currents are called eddy currents, or Foucault currents.
The power losses associated with eddy currents are evaluated through equation (2.16), which shows that Foucault losses depend on the squared intensity of the electric induction field, which, in turn, depends on the time derivative of $\vec{B}$. Naturally, the faster the variation of $\vec{B}$, the more important the losses.

### 2.1.6. Permanent magnets

In the context of electromechanical energy conversion devices, magnetic materials take great preponderance. Through their use it is possible to obtain large magnetic flux densities with relatively low levels of magnetizing force, which, considering magnetic forces and energy density increase with increasing flux density, plays a significant role in the performance of energy-conversion devices. This capacity is used in transformers to maximize the coupling between the windings as well as to lower the excitation current required for transformer operation. More interestingly for the issue under analysis in this report is the ability of magnetic materials to constrain and direct magnetic fields in well-defined paths. In electric machinery, magnetic materials are used to shape the fields to obtain desired torque-production and electrical terminal characteristics [7].

Ferromagnetic materials are found to be composed of a large number of regions in which the magnetic moments of all the atoms are parallel, giving rise to a net magnetic moment for that domain. In an unmagnetized sample of material, the domain magnetic moments are randomly oriented, and the net resulting magnetic flux in the material is zero. When an external magnetizing force is applied to this material, the domain magnetic moments tend to align with the applied magnetic field. As a result, the domain magnetic moments add to the applied field, producing a much larger value of flux density than would exist due to the magnetizing force alone. As the magnetizing force is increased, this behavior continues until all the magnetic moments are aligned with the applied field. At this point they can no longer contribute to increasing the magnetic flux density, and the material is said to be fully saturated. In the absence of an externally applied magnetizing force, the domain magnetic moments naturally align along certain directions associated with the crystal structure of the domain, known as axes of easy magnetization. Thus if the applied magnetizing force is reduced, the domain magnetic moments relax to the direction of easy magnetism nearest to that of the applied field. As a result, when the applied field is reduced to zero, although they will tend to relax towards their initial orientation, the magnetic dipole moments will no longer be totally random in their orientation. Instead, they will retain a net magnetization component along the applied field direction (i.e. magnetic hysteresis). Due to this phenomenon the relationship between $\vec{B}$ and $\vec{H}$ for a ferromagnetic material is both nonlinear and multivalued [7]. After reaching saturation, it is not possible to demagnetize the material by progressively decreasing applied magnetic
field to zero, since the operating point does not descend to the origin. In fact, when $\vec{H} = 0$, a positive remanence induction $\|\vec{B}\| = B_R$ is still found. In order to make $\vec{B} = 0$ it is necessary to magnetize the material in the reverse direction with a coercive field $\|\vec{H}\| = -H_C$. When the reversal magnetic flux density is removed, the flux density returns to a point lower than $B_R$ according to a minor hysteresis loop. Thus, the application of a reverse field has reduced the remanence, or remanent magnetism. Reapplying magnetic field intensity will again reduce the flux density, completing the minor hysteresis loop by returning the core to approximately the same value of flux density as before. The minor hysteresis loop may usually be replaced with little error by a straight line called the recoil line. This line has a slope called the recoil permeability. As long as the negative value of applied magnetic field intensity does not exceed the maximum value corresponding to the point where the minor hysteresis loop intersects the hysteresis loop, the PM may be regarded as being reasonably permanent. If, however, greater negative field intensity is applied, the magnetic flux density will be reduced to a value lower than that at intersection point between loops. On the removal, a new and lower recoil line will be established [4]. Repeating the magnetization/demagnetization process slowly and alternately, swinging from positive saturation to negative saturation, the so-called hysteresis loop is obtained [6], as illustrated in Figure 2-8.

![Figure 2-8 - Typical shape of a hysteresis loop, where $H_C$ and $B_R$ respectively denote the coercive field and the remanence induction - Source: [6].](image)

What tells apart permanent magnets (hard magnetic materials) from other ferromagnetic materials (soft magnetic materials) is that, although both have high values of remanent magnetization, the former also has a large value of coercivity, which means that the magnetic field intensity required to reduce the its flux density to zero is also large, as shown in Figure 2-9.
The coercivity can be thought of as a measure of the magnitude of the MMF required to demagnetize the material, being also a measure of the capability of the material to produce flux in a magnetic circuit which includes an air gap [7].

2.2. Preponderance of current density

In an AFPM machine the stator winding is located in the air gap magnetic field. Energy generation is accomplished by applying movement to the rotor which, together with its attached magnets, originates current in the stator windings, as illustrated by Figure 2-10. The output power of the machine is function of both its torque magnitude $T$ and angular speed $\omega$, as shown in equation (2.22).

$$P = \frac{dW}{dt} = \frac{d}{dt} \left( \int \vec{F} \cdot d\vec{x} \right) = \frac{d}{dt} \left( \int \vec{F} \cdot Rd\vec{\theta} \right) = \frac{T \, d\theta}{dt} = T \omega$$  \hspace{1cm} (2.22)

Since wind generators are characterized by working at low speeds, a powerful machine necessarily implies high torques. As indicated in equation (2.11), the electromagnetic torque is directly proportional to the magnetic force. Recalling equation (2.9), this force depends on the stator current and on the air gap magnetic field. In order to increase the output power, the tangential force on the machine will have to be enhanced. From section 2.1.6 of this report, it is already known that the rotor’s magnetic field is naturally limited by the hysteresis loop of its permanent magnets. Consequently, the rated current is the key variable in achieving high power machines. As already seen in equation (2.16), high current densities give rise to significant losses in the stator conductors, making the study and calculation of such losses a decisive factor in the designing process of the generators addressed in this research.
2.3. Stator winding losses

Although the scope of this research could be summarized by equation (2.16) applied to stator conductors, for the sake of computational modeling, the armature winding losses may be divided in terms of resistive and rotational losses, depending on where the current is originated. The first is related with the ease in current flowing through the conductor, while the second refers to the consequences of having stator conductors exposed to an alternating magnetic field, i.e. eddy effects. In addition, if this exposition to the magnetic field is not equally distributed by the subconductors, the flux linked with each one of them differs from one to the other, which results in different induced voltages. Thus, as the strands are short-circuited somewhere in the coil (when there is only transposition in the active region short-circuit happens in the end region; but the coil may also be totally transposed), eddy circulating currents arise between the subconductors. This fact naturally alters the resistive losses.

Figure 2-10 - Lorentz Force on one phase. 1 – rotor. 2 – PM. 3 – stator winding. v – velocity. B – magnetic field. I – stator current.
2.3.1. Resistive losses

The phenomena to be discussed relate to distortions in the distribution of current-density over the cross-section of conductors. These effects may be divided into three classes. Firstly, the skin effect due to disturbance of current density in a conductor due to the alternating magnetic flux linked with the same. It may be regarded as due either to imperfect penetration of electric current into the conductor, or to the greater reactance of the central core of the conductor with respect to the surface layer, whereby the current density is less on the inside than on the outside. In a uniform solid round wire, the skin effect is symmetrical with respect to its axis, while in solid wires of other than circular form, the skin effect is, in general, dissymmetrical. Secondly, the spirality effect found in spiraled stranded conductors and due to the reactance of the spirals. Lastly, the proximity effect, found in parallel linear conductors of any cross sectional form when in proximity, owing to the alternating magnetic flux from one penetrating the other [8].

Figure 2-11 represents a conductor with $M$ subconductors of arbitrary cross section placed parallel to the $Z$ axis of the associated coordinate system. Subconductors are built from the same material. Moreover, they are isotropic (i.e. electric permittivity and magnetic permeability are uniform in all directions), homogeneous (i.e. same properties at each point of the medium) and nonmagnetic.

![Figure 2-11 - Geometric configuration of the subconductors - Source: [9].](image)

Taking Maxwell’s equation (2.1) for each subconductor, considering both the differential relation that states $\nabla \cdot (\nabla \times \Phi) = 0$ and $\vec{E} = -\vec{E}_{\text{rot}} + \vec{E}_{\text{rad}}$, and recalling equation (2.4), last equality from set of equations (2.23) is found.

\[
\nabla \times \vec{E}(\vec{x}, t) = -\frac{\partial \vec{B}(\vec{x}, t)}{\partial t}
\]

\[
\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B}(\vec{x}, t) = \nabla \times \vec{A}(\vec{x}, t)
\]

\[
\vec{J}(\vec{x}, t) = \sigma \vec{E}_0(t) - \sigma \frac{\partial \vec{A}(\vec{x}, t)}{\partial t}
\]  

(2.23)
\( t \) is the time array, \( \vec{x} \) is a generic vector in the \( XY \) plan, \( \vec{J} \) is the current density, \( \sigma \) is the electrical conductivity, \( \vec{E}_0 \) is the electric field applied to the subconductor (e.g. \( -\vec{E}_{rad} \)), and \( \vec{A} \) is the magnetic vector potential.

Disregarding the displacement current and endpoint effects (i.e. end winding region), and assuming an uniform \( \sigma \), then \( \vec{E}_0 \), \( \vec{A} \) and \( \vec{J} \) have only the longitudinal component (i.e. along \( Z \) direction) [9].

Since both fields and currents are periodic and, therefore, harmonic functions of time, it is possible to separate spatial and temporal dependencies in equation (2.23), considering that the system is in the steady state [9]. Therefore, phasor notation can be used and last equality from set of equations (2.24) is obtained.

\[
\begin{align*}
\vec{J}_{\text{ind}} &= -j\omega \sigma A, \\
\vec{J} &= \sigma \vec{E}_0 + \vec{J}_{\text{ind}}
\end{align*}
\]

(2.24)

It is also possible to express the magnetic vector potential in terms of the current density vector by means of Maxwell’s equation (2.3), as shown in set of equations (2.25).

\[
\begin{align*}
\nabla \times \vec{H} &= \vec{J} \Rightarrow \nabla \times \vec{B} = \mu \vec{J} \\
\nabla \times \left( \nabla \times \vec{A} \right) &= \mu \vec{J} \\
\nabla \left( \nabla \cdot \vec{A} \right) - \nabla \cdot \left( \nabla \vec{A} \right) &= \nabla \left( \nabla \cdot \vec{A} \right) - \nabla^2 \vec{A} = \mu \vec{J}
\end{align*}
\]

(2.25)

Assuming the Coulomb gauge condition \( \nabla \cdot \vec{A} = 0 \), \( -\nabla^2 \vec{A} = \mu \vec{J} \) is reached, which results in the component form expressed in set of equations (2.26).

\[
\begin{align*}
\nabla^2 A_x &= -\mu J_x \\
\nabla^2 A_y &= -\mu J_y \\
\nabla^2 A_z &= -\mu J_z
\end{align*}
\]

(2.26)

In fact, set of equations (2.26) is nothing less than Poisson's equation three times over, thus it is possible to immediately write its unique solutions, which are presented in set of equations (2.27).
\[ A_y = \frac{\mu}{4\pi} \int \frac{J_y}{r_y} dy \]
\[ A_x = \frac{\mu}{4\pi} \int \frac{J_x}{r_x} dx \]  \hspace{1cm} (2.27)
\[ A_z = \frac{\mu}{4\pi} \int \frac{J_z}{r_z} dz \]

These solutions can be recombined to form the single vector solution of equation (2.28).

\[ \vec{A} = \frac{\mu}{4\pi} \int \frac{\vec{J}}{r} dV \]  \hspace{1cm} (2.28)

Recalling equation (2.24), the magnetic vector potential in \((x, y)\) because of a filamentary current in \((\xi, \eta, z_0)\) can be written in scalar form as shown in equation (2.29) [9].

\[ J(x, y, z) = \sigma E_0 + \frac{\mu}{4\pi} \iiint \frac{-j\sigma\omega J(\xi, \eta)}{\left[ (x-\xi)^2 + (y-\eta)^2 + (z-z_0)^2 \right]^{3/2}} d\xi d\eta dz_0 \]  \hspace{1cm} (2.29)

The longitudinal component dependency can be surpassed by solving the integral along the length of the conductor. The volume integral turns into a surface integral as shown in equation (2.30).

\[ J(x, y) = \sigma E_0 + \alpha \iint J(\xi, \eta) \ln \left[ (x-\xi)^2 + (y-\eta)^2 \right]^{1/2} d\xi d\eta \]  \hspace{1cm} (2.30)

\( J \) and \( E_0 \) are complex quantities in the \( Z \) axis direction, while the constant \( \alpha \) is the eigenvalue of the integral equation defined by (2.31).

\[ \alpha = j f \sigma \mu \]  \hspace{1cm} (2.31)

\( j \) is the imaginary unit, \( f \) is the frequency of the field and \( \mu \) is the magnetic permeability of the subconductor.

Generalizing equation (2.30) to the whole conductor with its \( M \) subconductors, integral equation (2.32) is found for the generic subconductor \( p \).
\[
J^p(x, y) = \sigma E_0^p + \alpha \sum_{q=1}^{M} \int J^q(\xi, \eta) \ln \left[ \left( x - \xi \right)^2 + \left( y - \eta \right)^2 \right] \frac{1}{2} d\xi d\eta \quad (2.32)
\]

In order to adjust the expression to numerical computation, each cross section \( S_p \) is subdivided into \( N_p \) rectangular domains with area \( S_D = ab \), as illustrated in Figure 2-12. At this point, the hypothesis is that the current density is uniform within each domain [9].

![Figure 2-12 - Subdivision of subconductor’s cross section into small rectangles - Source: [9]](image)

Equation (2.32) can be split into integrals over each domain of the subconductor \( q \), being the result then integrated over a generic domain \( d \) of the subconductor \( p \), resulting in the current density in the domain \( d \) of the subconductor \( p \), as shown by equation (2.33).

\[
J^p_d = \sigma E_0^p + \frac{\alpha}{S_D} \sum_{q=1}^{M} \sum_{k=1}^{N_q} J^q_k \int \int \ln \left[ \left( x - \xi \right)^2 + \left( y - \eta \right)^2 \right] \frac{1}{2} d\xi d\eta dx dy \quad (2.33)
\]

The iterated integral depends only on the geometric quantities and is related to the concept of geometric mean distance, introduced by Maxwell in his Treatise on Electricity and Magnetism. The integral can be evaluated using the Cauchy principal value and equation (2.33) can be rewritten as equation (2.34).

\[
J^p_d = \sigma E_0^p + \alpha S_D \sum_{q=1}^{M} \sum_{k=1}^{N_q} J^q_k \ln G_{dk}^m 
\]

(2.34)

\( G_{dk}^m \) denotes the geometric mean distance between domain \( d \) of the subconductor \( p \) and domain \( k \) of the subconductor \( q \). Being the domains of rectangular shape, there are formulas to compute directly geometric mean distances [9]. Such study falls out of the scope of this report.

For all domains of every subconductor, equation (2.34) represents a system of algebraic linear equations that in matrix notation may be written as equation (2.35).
\[
\{I - \alpha K\} J = C
\]  

(2.35)

\(I\) is the identity matrix. \(K\) is a matrix formed by submatrices \(K_{pq}\) whose dimensions are the product between the number of domains of its position (e.g. submatrix \(K_{ij}\) is \(N_i \times N_j\)) and whose elements are given by equation (2.36). \(J\) is a column matrix formed by submatrices \(J_{p}\), whose elements are the current densities in the domains of the generic subconductor \(p\). The column matrix \(C\) is defined in the same way but elements of submatrices \(C_p\) are constant and given by \(C_p = \sigma E_0^p\).

\[
K_{dq}^{pq} = S_d \ln G_{dq}^{pq}
\]  

(2.36)

To solve system (2.35) the size \(M\) column matrix \(I_p\) must be defined, which generic element \(I_j^p\) is the total current in the subconductor \(p\), as shown by equation (2.37).

\[
S_D \sum_{j=1}^{N_p} J_j^p = I_T^p
\]  

(2.37)

With \(H = [I - \alpha K]^{-1}\), the relation given in (2.37) yields

\[
S_D \begin{bmatrix}
\sum H_{11} & \cdots & \sum H_{M1} \\
\vdots & \ddots & \vdots \\
\sum H_{1M} & \cdots & \sum H_{MM}
\end{bmatrix}
= \begin{bmatrix}
C_1 \\
\vdots \\
C_M
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
I_T^1 \\
\vdots \\
I_T^M
\end{bmatrix}
\]

(2.38)

\(\sum H_{pq}\) is the algebraic summation of all elements of the submatrix \(H_{pq}\).

Now the usual methods to solve sets of linear algebraic equations may be used to find \(J\). Firstly, matrices \(K\) and \(H\) have to be computed. Then, equation (2.38) is solved yielding matrix \(C\). Finally, by applying \(C\) in (2.35), the solution \(J\) is found.

Considering that cross sections delimiting unitary length of the subconductor are equipotential surfaces [9], the impedance matrix is given by equation (2.39).

\[
Z = \frac{1}{\sigma S_D} \begin{bmatrix}
\sum H_{11} & \cdots & \sum H_{M1} \\
\vdots & \ddots & \vdots \\
\sum H_{1M} & \cdots & \sum H_{MM}
\end{bmatrix}^{-1}
\]  

(2.39)
To compute the resistive losses the unitary resistance of each subconductor must be obtained by application of the operator \( \text{Re} \{ \} \) to the corresponding apparent impedance, expressed in equation (2.40).

\[
Z^p_A = \sum_{q=1}^{M} Z_{pq}^q \frac{I^q}{I^p}
\]  

(2.40)

Circulating currents

As a cost effective solution for eddy current losses reduction, parallel thin wires are used in every turn. However, this may create a new problem, which is unless a complete balance of induced EMFs among the individual conducting paths is achieved, a circulating current between any of these parallel paths may occur, causing circulating eddy current losses [10]. In this section theory that supports the hybrid method, used further on this report, for calculating circulating currents is presented.

The current within each strand is the sum of two components [11]:

i. input current (AC stator current) that flows uniformly into every strand;

ii. circulating current that differs from strand to strand which sums to zero over the cross-section of all the strands in one bar.

The circuit considered to represent the active region of the winding is shown in Figure 2-13.

\[ I \] is the stator current, while \( i_k + I/n \) is the strand current, with \( I/n \) being the fraction of stator current that flows uniformly into every strand and \( i_k \) the circulating current, that differs from strand to strand. Magnetic coupling between strands is accounted for by mutual-inductances \( L_{k,l} \), while self-inductances \( L_{k,k} \) account for the influence of the internal magnetic flux produced by the current \( I/n \). It should be noted that the magnetic coupling between different coils is assumed to be small and, therefore, is disregarded in this analysis. Finally, \( R_k \) is the resistance of the strand.

Figure 2-13 - Strand circuit - Source: [11] - edited by author.
Applying Ohm’s law to one strand the voltage along the strand $V_k$ is found, as shown in equation (2.41).

$$V_k = R_k(i_k + I/n) + \sum_{i=1}^{n} L_{k,i} \frac{d(i_j + I/n)}{dt}$$  \hspace{1cm} (2.41)

Applying Faraday’s law of induction to one strand, equation (2.42) is found.

$$\oint E \cdot ds = -\frac{d\psi_s}{dt} \Rightarrow V_k - V = -\frac{d\psi_s}{dt}$$  \hspace{1cm} (2.42)

$V$ is the unknown voltage along the winding with respect to the fictitious conductor that, along with current filament of strand, closes path $s$.

### 2.3.2. Rotational losses

Recalling Maxwell’s equation (2.1), components of the magnetic field present in the air gap originate an electric field with the orientation illustrated in Figure 2-14. Since it is inside a conductive material, this electric field will then give rise to eddy current loops inside the conductor. As it can be seen in Figure 2-14, this current loops influence each other. For instance, top and bottom surfaces from the red loop have the same direction of lateral surfaces of the blue one, which means the resulting current in the conductor is function of both tangential and axial flux. This implies losses cannot be computed in separate considering components of magnetic induction independently of each other as it is done in [10]. The only option is to consider the system as a whole. Using the formulation presented in Appendix A, the relationship between the components of magnetic induction vector and the components of electric field vector is as shown in set of equations (2.43).
\[
\frac{\partial B_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \\
\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\
\frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y}
\]

(2.43)

Figure 2-15 illustrates the relationship between the dimensions of subconductor. It is clear that winding’s length is much bigger than both winding’s width and thickness. This type of geometry allows it to be assumed that the variation of component \( z \) of the electric field along \( X \) is small when compared to the variation of component \( x \) of the electric field along \( Z \). Likewise, the variation of component \( y \) of the electric field along \( X \) is small when compared to the variation of component \( x \) of the electric field along \( Y \). Thus, the problem can be reduced to system (2.44).

\[
\frac{\partial B_x}{\partial t} = \frac{\partial E_y}{\partial z} \\
\frac{\partial B_y}{\partial t} = -\frac{\partial E_z}{\partial z} \\
\frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y}
\]

(2.44)

As it stands, there is still interdependence between the components of magnetic induction and the system remains too complex to be solved because there is no information regarding the charge density inside the conductor. However, in an AFPM machine the longitudinal component of the magnetic flux is almost inexistent.
and can be disregarded when compared to the axial and tangential components. This acceptable approximation greatly simplifies the problem, allowing rotational losses to be calculated directly from equation (2.16), provided the electric field inside the conductor is easily obtained by solving system of partial differential equations (2.45).

\[
\frac{\partial E_y}{\partial t} = -\frac{\partial B_z}{\partial z} \\
\frac{\partial B_z}{\partial t} = \frac{\partial E_y}{\partial y} 
\]

(2.45)

Using phasor notation, \( B_y = B_{ym}e^{j\omega t}e^{j\phi_x} = \overline{B_y}e^{j\phi_x} \), \( B_z = \overline{B_z}e^{j\phi_x} \) and \( E(y, z) = \overline{E}e^{j\phi_x} \), the solution is found in last equality of set of equations (2.46).

\[
\begin{align*}
  j \omega \overline{B}_y &= -\frac{\partial \overline{E}}{\partial z} \Rightarrow \overline{E} = -j \omega \overline{B}_y + f(y) + C \\
  j \omega \overline{B}_z &= \frac{\partial f(y)}{\partial y} \Rightarrow f(y) = j \omega y \overline{B}_z \\
  \overline{E} &= j \omega y \overline{B}_z - j \omega \overline{B}_y + C
\end{align*}
\]

(2.46)

To determine constant \( C \), the constitutive relation that states \( \nabla \cdot \vec{J} = 0 \) must be considered. Using Gauss’ theorem, \( \int_{S_y} \vec{J} \cdot \overrightarrow{n} dS = 0 \) is obtained, which means that the number of \( \vec{J} \) lines entering a given volume is equal to those leaving it. Basically, this fact indicates that current loops flow around a symmetry axis, as illustrated in Figure 2-16, having the current density symmetric values with respect to it. Since \( \vec{J} \) is directly proportional to \( \overline{E} \), as already seen in equation (2.4), the boundary condition for the coordinate system adopted in Figure 2-16 is \( \overline{E}(y = 0, z = 0) = 0 \), which implies \( C = 0 \). With the electric field defined, the total rotational losses in the conductor is simply found using equation (2.16), as shown in equation (2.47), where \( c \) is the length of the conductor.

\[
\Delta P_c = \int_V \sigma E^2 dV = \int_V \sigma \left( \frac{j \omega (\overline{y} \overline{B}_z - z \overline{B}_y)}{\sqrt{2}} \right)^2 dV = c \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \sigma \left( \frac{j \omega (y \overline{B}_z - z \overline{B}_y)}{\sqrt{2}} \right)^2 dydz (2.47)
\]
2.4. The finite element method

The two most popular methods of deriving the finite element equations are the variational approach and the Galerkin approach, which is a special case of the method of weighted residuals (MWR). The variational method was first applied to problems in magnetics and occupies a large part of the early literature. However, there are a number of practical cases in machinery analysis in which the variational expression either is not known or does not exist and where the weighted residual method can be applied. Contrarily, due to its greater generality, the Galerkin approach has increased in popularity [12] and is considered in this report.

The MWR may be applied by beginning with the operator equation \( \Gamma(x) = 0 \) on region \( \Omega \) with boundary conditions on the boundary \( C \). Substituting an approximate solution \( x \) into the operator function, a residual \( R \) is obtained since \( x \neq x \), as shown in equation (2.48).

\[
\Gamma(x) = R
\] (2.48)

The MWR now requires that the integral of the projection of the residual on a specified weighting function is zero over the domain of interest. The choice of the weighting function determines the type of MWR. The Galerkin method implies choosing the weighting function to have the same form as the finite element shape function [12], as it will be seen further on this analysis.

Figure 2-16 - Symmetry axis of the current density within a cross section of the conductor.
The time harmonic form of the diffusion equation with $\vec{A}$ as the unknown will be used as example. Such equation was deduced in last equality of set of equations (2.25). For a linear two dimensional Cartesian problem, the Laplacian on the left-hand side of the diffusion equation develops as shown in equation (2.49).

$$\frac{1}{\mu} \nabla^2 \vec{A} = \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial y^2} = -\vec{J}_0 + j\omega\sigma\vec{A} \quad (2.49)$$

As shown in equation (2.50), substituting $\vec{A}$ for an approximation $\hat{A}$, a residual $R$ is obtained.

$$R = \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial y^2} + \vec{J}_0 - j\omega\sigma\hat{A} \quad (2.50)$$

Now, as done in equation (2.51), the residual must be pondered with a weighting function $W$ and the integral over the region set to zero.

$$\iint_{\Omega} RWdxdy = 0 \Rightarrow$$

$$-\iint_{\Omega} W \left( \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial y^2} \right)dxdy + j\omega\sigma\iint_{\Omega} WAdxdy = \iint_{\Omega} W\vec{J}_0dxdy \quad (2.51)$$

Recalling that integration by parts states that $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$, by making $v' = \left( \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial y^2} \right)$ and $u = W$, first term of last equality of equation (2.51) can be integrated as shown in equation (2.52), where the last term is on the boundary $C$ with $n$ being the outward normal unit vector.

$$-\iint_{\Omega} W \left( \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial y^2} \right)dxdy =$$

$$\iint_{\Omega} \frac{1}{\mu} \left( \frac{\partial W}{\partial x} \frac{\partial \vec{A}}{\partial x} + \frac{\partial W}{\partial y} \frac{\partial \vec{A}}{\partial y} \right)dxdy - \oint_{C} \frac{1}{\mu} W \frac{\partial \vec{A}}{\partial n}dc \quad (2.52)$$

Substituting equation (2.52) into equation (2.51), it is possible to break the surface into summations over small areas, as shown in equation (2.53). For instance, the surface may be meshed with $M$ triangles (the finite elements) and replace the integral over the entire domain with the summation of the integral over the individual triangles.
\[
\sum_{e=1}^{M} \left\{ \frac{1}{\mu'} \int_{\partial e} \left[ \frac{\partial W^e}{\partial x} \frac{\partial A^e}{\partial x} + \frac{\partial W^e}{\partial y} \frac{\partial A^e}{\partial y} \right] \, dx \, dy \right\} + \int_{\Gamma} \left( \omega \sigma \int_{\partial e} W^e A^e d\Gamma - \frac{1}{\mu'} \frac{\partial A^e}{\partial n} \oint_{c} W^e \, ds \right) = J^0 \int_{\Omega} W^e \, dx \, dy
\]  

(2.53)

The line integral in equation (2.53) only needs to be evaluated over elements which have a side in common with the boundary of the problem. Usually, this integral is simply set to zero implying that \( \frac{\partial A}{\partial n} = 0 \) which results in the so called natural boundary condition. However, this integral is often used in problems in which the finite element method is coupled with other solution techniques (e.g. air gap elements calculations). In this case the integral must be evaluated. Considering a triangular element with vertices \( i, j, k \), at coordinates \( (x_i, y_i) \), \( (x_j, y_j) \), \( (x_k, y_k) \), respectively. The vertices are nodes at which the unknown vector potentials will eventually be calculated. Assuming the potential varies linearly in the element, a linear or first order element is obtained. With this approximation, it is possible to express the vector potential at any point in the triangle as shown in equation (2.54).

\[
\vec{A} = C_1 + C_2 x + C_3 y
\]

(2.54)

\( C_1 \), \( C_2 \) and \( C_3 \) are constants to be determined. It shall be noted that since the vector potential varies linearly, the flux density, which is the derivative of the potential, is constant in the triangle. At point \( (x_i, y_i) \), \( \vec{A} \) must be equal to \( \vec{A}_i \), with the same happening for nodes \( j \) and \( k \). Thus, system of equations (2.55) is obtained, which shall be solved for \( C_1 \), \( C_2 \) and \( C_3 \) in terms of the potential and geometry using Cramer’s rule. The results are expressed in set of equations (2.56).

\[
\begin{align*}
\vec{A}_i &= C_1 + C_2 x_i + C_3 y_i \\
\vec{A}_j &= C_1 + C_2 x_j + C_3 y_j \\
\vec{A}_k &= C_1 + C_2 x_k + C_3 y_k
\end{align*}
\]

(2.55)
It is verified that the determinant in the dominator is equal to twice the area of the triangle $S_\lambda$. Using the results of set of equations (2.56) in equation (2.54), it is possible to express $\bar{A}$ as equation (2.57).

$$\bar{A} = \frac{\left( a_i + b_j x + c_k y \right) \bar{A}_i + \left( a_j + b_k x + c_i y \right) \bar{A}_j + \left( a_k + b_i x + c_j y \right) \bar{A}_k}{2S_\lambda}$$  \hspace{0.5cm} (2.57)

$a$, $b$, and $c$ are the determinants of the 2-by-2 matrices associated with $\bar{A}_n$ in set of equations (2.56). For instance, for $\bar{A}_i$, $a_i = x_j y_k - x_k y_j$, $b_i = y_j - y_k$, and $c_i = x_k - x_j$. The coefficients of the nodal potentials in equation are called shape functions. The potential can be expressed as the sum of the shape function times the nodal potential, as shown in equation (2.58), where $m$ is the number of nodes in the element and $N_i$ is the shape function associated with $\bar{A}_i$.

$$\bar{A} = \sum_{i=1}^{m} N_i(x, y) \bar{A}_i$$  \hspace{0.5cm} (2.58)

Equation (2.59) shows the potential in the element expressed in matrix form.
\[
\begin{pmatrix}
A^e & (N_i^e & N_j^e & N_k^e)
\end{pmatrix}
\begin{pmatrix}
A^e
\end{pmatrix}
\]

The shape function \( N_{e} \) is equal to \( (a^n_{x} + b^n_{x}x + c^n_{x}y) / (2S_{\Delta}) \). In the Galerkin Method the weighting function is chosen to be the same as the shape function. Thus, equation (2.60) is obtained.

\[
W^e = \begin{pmatrix}
N_i^e \\
N_j^e \\
N_k^e
\end{pmatrix}
\]

Taking derivatives with respect to \( x \) and \( y \), sets of equations (2.61) and (2.62) are obtained.

\[
\frac{\partial A^e}{\partial x} = \frac{1}{2S_{\Delta}} \begin{pmatrix} b_i^e & b_j^e & b_k^e \end{pmatrix}
\begin{pmatrix}
A^e
\end{pmatrix}
\]

\[
\frac{\partial A^e}{\partial y} = \frac{1}{2S_{\Delta}} \begin{pmatrix} c_i^e & c_j^e & c_k^e \end{pmatrix}
\begin{pmatrix}
A^e
\end{pmatrix}
\]

\[
\frac{\partial W^e}{\partial x} = \frac{1}{2S_{\Delta}} \begin{pmatrix} b_i^e \\
b_j^e \\
b_k^e \end{pmatrix}
\]

\[
\frac{\partial W^e}{\partial y} = \frac{1}{2S_{\Delta}} \begin{pmatrix} c_i^e \\
c_j^e \\
c_k^e \end{pmatrix}
\]

Noting that the derivatives are constants (i.e. \( x \) and \( y \) independent), and that \( \int_{\Omega} dx dy = S_{\Delta} \), first and second terms of equation (2.53) turn into equation (2.63) and (2.64), respectively.
The coefficient matrices in equations (2.63) and (2.64) are usually referred to as the stiffness matrix and the mass matrix, respectively. Finally, the right-hand side of equation (2.53) becomes equation (2.65), where 

\[ \bar{x} = \frac{1}{3}(x_i + x_j + x_k) \quad \text{and} \quad \bar{y} = \frac{1}{3}(y_i + y_j + y_k) \]

are the coordinates of the centroid of the triangle.

\[
\frac{1}{\mu'} \iint_{\Omega'} \frac{\partial W^{\varepsilon}}{\partial x} \frac{\partial A^c}{\partial x} + \frac{\partial W^{\varepsilon}}{\partial y} \frac{\partial A^c}{\partial y} \, dxdy =
\]

\[
\frac{1}{\mu'} \left( \frac{\partial W^{\varepsilon}}{\partial x} \frac{\partial A^c}{\partial x} + \frac{\partial W^{\varepsilon}}{\partial y} \frac{\partial A^c}{\partial y} \right) \iint_{\Omega'} dxdy =
\]

(2.63)

\[
\frac{1}{4\mu' S_\Delta} \left( \begin{array}{ccc}
    b_i^2 + c_i^2 & b_i b_j^e + c_i c_j^e & b_i b_k^e + c_i c_k^e \\
    b_j^e b_i^e + c_j^e c_i^e & b_j^e b_j^e + c_j^e c_j^e & b_j^e b_k^e + c_j^e c_k^e \\
    b_k^e b_i^e + c_k^e c_i^e & b_k^e b_j^e + c_k^e c_j^e & b_k^e b_k^e + c_k^e c_k^e \\
\end{array} \right) \begin{pmatrix}
    A_i^c \\
    A_j^c \\
    A_k^c \\
\end{pmatrix}
\]

(2.64)

\[
 j \omega \sigma^e \iint_{\Omega'} W^c A^c dxdy =
\]

\[
 j \omega \sigma^e \iint_{\Omega'} \begin{pmatrix}
    N_i^e \\
    N_j^e \\
    N_k^e \\
\end{pmatrix} \begin{pmatrix}
    N_i^e & N_j^e & N_k^e \\
\end{pmatrix} \begin{pmatrix}
    A_i^c \\
    A_j^c \\
    A_k^c \\
\end{pmatrix} dxdy =
\]

\[
 j \omega \sigma^e S_\Delta \begin{pmatrix}
    2 & 1 & 1 \\
    1 & 2 & 1 \\
    1 & 1 & 2 \\
\end{pmatrix} \begin{pmatrix}
    A_i^c \\
    A_j^c \\
    A_k^c \\
\end{pmatrix}
\]

\[
\overline{J}_0' \iint_{\Omega'} W' dxdy = \overline{J}_0' \iint_{\Omega'} \begin{pmatrix}
    (a_i^e + b_i^e x + c_i^e y)/(2S_\Delta) \\
    (a_j^e + b_j^e x + c_j^e y)/(2S_\Delta) \\
    (a_k^e + b_k^e x + c_k^e y)/(2S_\Delta) \\
\end{pmatrix} dxdy =
\]

(2.65)

\[
\overline{J}_0' \begin{pmatrix}
    (a_i^e + b_i^e \bar{x} + c_i^e \bar{y})/2 \\
    (a_j^e + b_j^e \bar{x} + c_j^e \bar{y})/2 \\
    (a_k^e + b_k^e \bar{x} + c_k^e \bar{y})/2 \\
\end{pmatrix} = \frac{\overline{J}_0' S_\Delta}{3} \begin{pmatrix}
    1 \\
    1 \\
\end{pmatrix}
\]
Once the element matrices are found for each element they are used to form the global matrix in a process known as assembly. Each element matrix has rows and columns corresponding to the nodes in the element. In the assembly process all of the element matrices are simply added together to form the global matrix. For a problem with \( m \) nodes the start is an \( m \times m \) zero matrix. Then, each element is passed through and \( ij \) terms of the stiffness and mass matrices are added to the corresponding \( ij \) term in the global matrix [12].

To obtain a unique solution to the problem, either the unknown or its normal derivative must be specified at each point on the boundary. Furthermore, the potential of at least one point in the problem must be specified to render the global matrix non-singular. To specify the potential at a point is called the Dirichlet condition. If the potential is specified as a constant along a line (e.g. the boundary), this becomes an equipotential. The homogeneous Dirichlet condition implies that the value of the specified potential is set zero. On the other hand, if the normal derivative of the potential is specified instead, it is called a Neumann condition, being the homogeneous Neumann condition \( \frac{\partial \phi}{\partial n} = 0 \), also called the natural boundary condition [12]. Such condition was obtained by default as a result of discarding the surface integral term in equation (2.53).

The formulation and example given in this section were linear. In the analysis of electrical machines the problems are almost always nonlinear due to the presence of ferromagnetic materials. Moreover, good designs will typically operate near the saturation point. The magnetic permeability is nonhomogeneous and will be a function of the local magnetic fields which are unknown at the start of the problem. Since the permeability appears in all of the element stiffness matrices, an iterative process must be used to keep correcting the permeability until it is consistent with the field solution. A simple method is beginning by assuming permeability for each element in the mesh. The problem is solved and the magnitude of the flux density in each element computed. The permeabilities are then corrected so that they are consistent with the computed values of flux density, being the problem solved again with new flux densities being found and permeabilities corrected again. The process continues until the results stop changing, i.e. until the change is smaller than a specified value. The most popular method of dealing with nonlinear problems in magnetics is the Newton-Raphson method [12].

### 2.5. Approach

Since the large scale wind generator under analysis works at low frequencies, resistance limited approach is used to evaluate eddy current losses [13], which implies the following assumptions:

i. as conductor dimensions are small, flux produced by eddy currents has a negligible influence on the total field [14];

ii. conductor dimensions are smaller than the skin depth: the eddy current losses due to the load current is ignored, as it is only a very small percentage of the losses due to the air gap magnetic field [14].
Both solutions presented in this study concern using parallel wires with smaller cross sections instead of one thick conductor. Attending to Faraday’s law of induction, this may create a new problem: unless a complete balance of induced EMFs among the individual conducting paths is achieved, a circulating current between any of these parallel paths may occur, causing circulating eddy current losses [4]. Basically, not only time derivative of flux linkage due to rotor magnets’ movement is different from strand to strand, but also time derivative of flux linkage in each strand due to current that flows in every other strand also differs from one strand to another. By transposing the wires in such a way that each parallel conductor occupies all possible layer positions for the same length of the coil, the induced EMFs in all parallel conductors are equalized and circulating currents can generally be ignored in a heavily twisted coil, which is the case of the solutions presented in this paper (i.e. 360 degrees transposition) [4].

Alternating current that flows in stator conductors originates magnetic field that alters the one coming from the rotor, as seen in Ampère’s law equation (2.18), illustrated in Figure 2-17. The influence of this armature reaction on the eddy current losses is usually insignificant [4], and is therefore disregarded in this analysis. However, it is intrinsically considered in the Joule losses analysis, where the magnetic field created by the current in one strand influences the impedance of the remaining strands, as it will be seen further on this report.

![Figure 2-17 - Application of Ampère’s law to the current flowing threw a cylindrical conductor - Source: [6].](image)

The most straightforward approach to solve the problem would be to completely model the whole segment of the machine, with the transposed coils in between the double-sided rotor. Then to run a transient simulation on the model by rotating the segment of the rotor at rated speed. Such approach would give an exact replica of the real world environment, and losses would be computed considering circulating currents, rotational and resistive losses all together. However, several problems arise when trying to perform this method. Firstly, designing the end-winding region where different turns connect is a task of great complexity. Since the behavior of a transposed winding is to be evaluated, the simple method of designing a phase coil as a massive cross section conductor (and simply input to the software the number of turns in play for it to compute internally) does not work. It is necessary to model and transpose each turn individually, which means the coil has to be built by connecting turn by turn at the end winding region. Although several methods were attempted in order to achieve a satisfactory model of one coil, intersections between different strands kept occurring. Of course it would have been possible to turn to a more powerful three-dimensional software (e.g. SolidWorks) just for designing
purposes and then convert it into a Maxwell model, but two reasons advised to search for other solutions: 1. the idea of testing cases with several situations (i.e. several number of strands and different types of winding) required the coding to be kept only about the Maxwell environment; 2. analysis were performed on the active region devoid of the end-winding region, and a considerably amount of time was consumed just to compute the initial mesh, so, even considering a good design of the coil, with the end region, was achieved, simulation would most likely not return any results due to lack of computational power. Moreover, the method followed to calculate eddy current losses allows a solution to be found in a magnetostatic environment, since field periodically variation can be computed using Fourier analysis along pole-pitch. To run a transient solution type would not only be an uncertain step considering the complexity of the design itself, but would also produce losses results that would not allow distinctions to be made between resistive and rotational losses. Consequently, it would not be possible to depict losses sources of each solution and understand what can be changed to achieve a more competitive winding.

Considering the above, rotational and resistive losses are studied separately with the influence of circulating currents also being considered in a dedicated script and only then included as part of resistive losses.
Chapter 3

Simulation

The goal of this chapter is to describe in detail the computational methods used to calculate stator winding losses. Firstly, the three-dimensional model of the segment used to study the machine is presented, with special emphasis being given to the architecture of the two transposed winding technologies under analysis. Results for resistive and rotational losses are presented and the critical aspects behind the methods’ algorithms discussed.


### 3.1. Computational model of the machine

Table 3-1 indicates the specifications of the machine.

#### Table 3-1 - Machine specifications.

<table>
<thead>
<tr>
<th></th>
<th>Nominal power [MW]</th>
<th>Outer diameter [m]</th>
<th>Inner diameter [m]</th>
<th>Number of poles</th>
<th>Air gap distance between surface of winding and permanent magnets [mm]</th>
<th>Magnet thickness [mm]</th>
<th>Iron thickness [mm]</th>
<th>Number of turns</th>
<th>Winding width [mm]</th>
<th>Winding thickness [mm]</th>
<th>Magnet pitch factor</th>
<th>Effective value of stator rated current [A]</th>
<th>Rated speed [rpm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>24.8</td>
<td>22.853</td>
<td>360</td>
<td>37.2</td>
<td>23.6334</td>
<td>21.1022</td>
<td>38</td>
<td>65.346</td>
<td>22.268</td>
<td>0.63</td>
<td>97.197</td>
<td>12</td>
</tr>
</tbody>
</table>

#### 3.1.1. Stator winding

The armature of the machine is composed by nonconductive material that supports phase current conductors. Instead of a single massive cross section, each coil turn is built from several sub conductors of rectangular cross section. The model under analysis consists of several strands placed in two adjacent rows. In order to reduce losses due to circulating currents, 360 degrees transposition is applied in the active region of the machine. The main difference between the two types of transposed wire taken into account is illustrated in Figure 3-1. Basically, while transposition process in CTC is physically stepped, i.e. each strand shifts position along a small slant and then develops its path perpendicularly to coil cross section plan, in Roebel each strand extends itself obliquely to coil cross section plan.

![Figure 3-1](image)

**Figure 3-1 - 2-D profile comparison between CTC and Roebel.**

The conductive material is copper with properties given on Table 3-2. The filling factor considered is 0.5, which means that each cross section of one strand has an inner copper surface surrounded by a vacuum ring of equal area.

#### Table 3-2 - Copper material properties.

<table>
<thead>
<tr>
<th>Relative permittivity</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative permeability</td>
<td>0.9999991</td>
</tr>
<tr>
<td>Bulk conductivity [S.m⁻¹]</td>
<td>58E6</td>
</tr>
</tbody>
</table>
Figure 3-2 illustrates the sizing of a CTC. Being $T$, $W$ and $L$ the thickness, width and length, respectively, of one turn of the winding without applied transposition, $t = T/(n+1)$, $w = W/2$ and $l = L$ are the thickness, width and length, respectively, of each CTC’s strand with $n$ sub conductors per row. The extra space needed in height is to allow shifting between rows. In a 360 degrees transposition, the length each strand stands in one of the $2n$ possible positions (or the transposition pitch) is $dx = l/(2n)$. Longitudinal distance between the beginning/end of a slope and the transposition pitch is $delta = dx/10$. During a shift in the same row, a strand travels $dz = delta/2$ along the winding length direction, while a shift between rows means a $dy = dx/5$ displacement in the same direction. Naturally, shifts in the same row mean a displacement of $dz = t$ along axial direction of the machine, while a shift between rows entails a $dy = w$ displacement along the winding width direction.

Figure 3-2 - CTC with 3 strands per row 360 degrees transposed.

Figure 3-3 illustrates a Roebel bar. Same logic of CTC is applied, except thickness of each strand is $t = T/(n+2)$, being the space lost to allow shifting between rows one strand’s height longer than CTC. This is due to the fact of each strand displacing itself obliquely. During a shift between rows, a strand travels $t$ upwards and downwards shifting sideward in between. Each of these three displacements entails a longitudinal progression of $dx/3$.
3.1.2. Rotor

Figure 3-3 - Roebel with 3 strands per row 360 degrees transposed.

Figure 3-4 illustrates one segment of the rotor of the machine containing one pole pair. Permanent magnets are made of N42SH with material properties described in Table 3-3, while rotor steel is made of DW465-50 with material properties listed in Table 3-4 and hysteresis loop represented in Figure 3-5.

Table 3-3 - N42SH material properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative permeability</td>
<td>1.05</td>
</tr>
<tr>
<td>Bulk conductivity [S.m⁻¹]</td>
<td>560E3</td>
</tr>
<tr>
<td>Magnetic coercivity along Z [A.m⁻¹]</td>
<td>987E3</td>
</tr>
</tbody>
</table>
3.2. Resistive losses

The goal is to determine stator’s winding conductive performance under a range of frequencies around the rated speed of the machine, so that some understanding can be gained about the influence subconductors have on each other’s resistance. To account for the skin effect, a two-dimensional model is run for only one straight subconductor with the same cross section dimension it would have on a transposed arrangement. Then, the proximity effect is evaluated by running a two-dimensional model in a stranded winding without applied transposition. It should be noted that when studying the proximity effect, the skin effect is automatically considered. Finally, the spirality effect is accounted for by running a three-dimensional simulation on the transposed winding. Naturally, the skin and proximity effects are intrinsically considered. As already discussed in section 2.5 of this report, the analysis is done solely considering the winding itself (i.e. rotor’s magnetic field is disregarded). The software runs an internal algorithm based on the method described in subsection 2.3.1 of this report, being the most significant difference the fact that the geometric mean distances have to be considered in a three-dimensional environment to accurately account for the spirality effect.

From the user’s point of view, the procedure followed to determine conductor’s resistance is straightforward and spares both parallel and post calculations on the data produced by software analysis. Basically, after drawing the winding, it is only necessary to assign stator current and an impedance matrix to subconductors before running the simulation. However, this does not mean that the calculations are light. In fact the computational effort needed to determine the stator’s winding resistance on the three-dimensional model is quite heavy since a detailed enough interior mesh is necessary inside of each subconductor.
3.2.1. Resistance

Joule losses results are presented below. It should be noted that tables with the resistance for each frequency indicate results with figures quite further the significant ones. Naturally, these results are not that rich in terms of detail. Software internal calculations were performed admitting one percent energy error. As there is direct proportionality between power and energy, results are only meaningful until the hundredth of mOHM. Consequently, as it will be seen in the results, it can be stated that for the array of frequencies under study, the winding is working under its DC resistance.

To evaluate solely the skin effect on each strand, it is necessary to study the latter isolated. Thus, tests were performed on a straight conductor with the same cross section of one strand for every transposition type in study. It was verified that, regardless of the technology, there is no measurable difference between the resistance of one isolated strand and the resistance of the same subconductor performing in a stranded wire. In other words, for the small dimensions and the array of low frequencies in study, the skin and proximity effects play the same role in shaping the resistive losses of the wire. As a result, and to avoid an unnecessary and monotonous repetition, resistances obtained from the skin effect simulations were suppressed from this report.

To serve the purpose of evaluating the effect of subdividing of the conduction path into several strands, it is necessary to test the behavior of the massive cross section that constitutes one turn not stranded. Figure 3-6 illustrates the distribution of the magnitude of current density vector over the cross section when performing under the rated frequency.

![Figure 3-6 - Distribution of the magnitude of current density vector at 36 Hz over the cross section of a nonstranded conductor.](image)

Table 3-5 lists, for each frequency, the electrical resistance of the conductor.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Massive section</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.876640</td>
<td>0.876652</td>
<td>0.876673</td>
<td>0.876709</td>
<td>0.876773</td>
<td>0.877171</td>
<td>0.877832</td>
</tr>
</tbody>
</table>

Tests performed for a CTC with two strands per row are described below. Same procedure was followed to the remaining technologies and results submitted to Appendix B. Conclusions deducted here are extensible to every technology. To evaluate the proximity effect, a stranded nontransposed winding is tested. Figure 3-7 illustrates the distribution of the magnitude of current density vector over the cross section when performing under the rated frequency.

Figure 3-7 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed CTC with 2 strands per row.

It is verified that, although a rearrangement in the current density is noticeable, the differences are not big enough to cause a measurable change in the resistance of the conductor, as shown in Table 3-6 that lists, for each frequency, the electrical resistance of each strand. The increase with respect to the massive cross section case is due to the fact that extra space, of the size of one strand’s height, is needed to allow the transposition to be physically executable.

Table 3-6 - Resistance [mOhm] of each strand in a non-transposed CTC winding with 2 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3, 4</td>
<td>5.259841</td>
<td>5.259851</td>
<td>5.259869</td>
<td>5.259900</td>
<td>5.259955</td>
<td>5.260296</td>
<td>5.260866</td>
</tr>
</tbody>
</table>

To evaluate the spirality effect, a stranded and transposed winding is tested using a three-dimensional model. Figure 3-8 shows the distribution of the magnitude of current density vector over the cross sections when performing under the rated frequency. Table 3-7 lists, for each frequency, the electrical resistance of each strand.

Table 3-7 - Resistance [mOhm] of each strand in a CTC winding with 2 strands per row.

<table>
<thead>
<tr>
<th>Strand</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.272420</td>
<td>5.272422</td>
<td>5.272426</td>
<td>5.272433</td>
<td>5.272446</td>
<td>5.272523</td>
<td>5.272652</td>
</tr>
<tr>
<td>2</td>
<td>5.272520</td>
<td>5.272522</td>
<td>5.272526</td>
<td>5.272533</td>
<td>5.272546</td>
<td>5.272624</td>
<td>5.272754</td>
</tr>
<tr>
<td>3</td>
<td>5.272568</td>
<td>5.272571</td>
<td>5.272575</td>
<td>5.272582</td>
<td>5.272594</td>
<td>5.272672</td>
<td>5.272802</td>
</tr>
<tr>
<td>4</td>
<td>5.272500</td>
<td>5.272503</td>
<td>5.272507</td>
<td>5.272514</td>
<td>5.272526</td>
<td>5.272604</td>
<td>5.272734</td>
</tr>
</tbody>
</table>
Figure 3-8 - Distribution of the magnitude of current density vector over a CTC with 2 strands per row.

It is verified that the distribution of current density remains the same as in the nontransposed case and that the resistance raise is only due to the winding length increasing, which naturally occurs as a result of the interlacement.

**High frequency**

Although the machine rotates at low speeds, the grid connection process involves electronic converters, which generally work at higher frequencies, introducing high order current harmonics in the stator. In order to evaluate this effect, each of the transposed technologies was studied under frequencies of 1 kHz, 10 kHz and 100 kHz. Results are presented in Table 3-8. As expected, the resistance of each subconductor increases with the frequency, and is no longer approximated by the DC resistance. However, the increment is only measurable in the hundredth of kHz and even that is not drastic enough to make these solutions useless for higher speeds applications. Other conclusion to be pointed out is that, contrarily to what is expected in high frequency systems with parallel wires, there still is no significant difference between the impedance of each subconductor, proving the utility of the transposition.

<table>
<thead>
<tr>
<th>Winding type</th>
<th>Low frequency [mOhm]</th>
<th>1 kHz [p.u.]</th>
<th>10 kHz [p.u.]</th>
<th>100 kHz [p.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC – 4 strands</td>
<td>1.32</td>
<td>1</td>
<td>1</td>
<td>1.12</td>
</tr>
<tr>
<td>CTC – 6 strands</td>
<td>1.17</td>
<td>1</td>
<td>1</td>
<td>1.09</td>
</tr>
<tr>
<td>CTC – 8 strands</td>
<td>1.10</td>
<td>1</td>
<td>1</td>
<td>1.07</td>
</tr>
<tr>
<td>CTC – 10 strands</td>
<td>1.06</td>
<td>1</td>
<td>1</td>
<td>1.05</td>
</tr>
<tr>
<td>Roebel – 4 strands</td>
<td>1.76</td>
<td>1</td>
<td>1</td>
<td>1.05</td>
</tr>
<tr>
<td>Roebel – 6 strands</td>
<td>1.46</td>
<td>1</td>
<td>1</td>
<td>1.05</td>
</tr>
<tr>
<td>Roebel – 8 strands</td>
<td>1.32</td>
<td>1</td>
<td>1</td>
<td>1.04</td>
</tr>
<tr>
<td>Roebel – 10 strands</td>
<td>1.23</td>
<td>1</td>
<td>1</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Litz wire

It is very difficult, if not impossible, to build a three-dimensional model of a Litz wire. Even if a draw was achieved, the software would most likely not be able to cope with the large number of subconductors involved and, most important, with their high degree of transposition. However, in order to have some term of comparison between the transposed winding modeled in this report and the more conventional Litz wire solution, the proximity effect test described before was applied to one turn of the machine. Basically, sets of circular subconductors with 0.25 mm of radius were spread over half surface of the cross section to account for the filling factor. This resulted in a model with 97 strands organized in horizontal rows of 5 or 4 subconductors. Figure 3-9 shows the distribution of the magnitude of current density vector over the cross section when performing under the rated frequency.

![Figure 3-9 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed Litz wire.](image)

As it can be seen, contrarily to what happens in the CTC and Roebel transpositions, there is no symmetry in the model, which results in every strand having its own unique resistance distinct from the other subconductors. The average resistance per strand at rated frequency is 0.0855217 Ohm, which means \( P_{\text{strand}} = R \cdot I^2 \approx 0.08587 \) W of Joule losses per strand or a total resistive losses of \( P_{\text{total strands}} = \text{Total strands} \cdot P_{\text{strand}} \approx 8.3 \) W in the modeled half turn, as circulating currents are completely absent in this kind of technology.

### 3.2.2. Circulating currents

Although it is simple to theoretical formulate the problem, performing in its completeness the task of determining the circulating currents between strands is not straightforward, due to the complexity of a design loyal to the real environment, and the subsequent heavy computational effort required. As already seen, circulating currents are mostly function of the induced voltage on each subconductor, which is directly related with the time derivative of the flux linked with each subconductor. The higher the differences between these induced voltages are the more significant circulating currents turn out to be. What happens in the real world is that a complete roll of winding is purchased to achieve a determined transposition degree in the active region. However, for price reasons, wire from the end winding region is also made of the same roll, which means that the strands are not short-circuited every half turn. In fact, the whole phase circuit is stranded even when it
travels from one coil to the other. In this analysis, only one half turn is considered, which means that the impedance of each strand disregards the influence of: 1. other turns in the same coil; 2. end winding region; and 3. surrounding coils. Naturally, the influence of these elements’ magnetic induction on the flux linkage of the half turn is also not considered. More importantly, the fact that each turn is short-circuited in the active region considerably decreases the voltage drops between strands, which means that results for the circulating currents presented next should not be seen quantitatively, but as a comparator between technologies instead, since the conclusions drawn are extensible to a real environment situation. As a result, the approach followed to solve the problem is to determine the time variation of the flux linked with each strand by rotating a segment of the rotor along its radial length. Then, as the obtained plots are periodic in terms of the whole machine, circulating currents can easily be determined by performing Fourier analysis on them and solving system described in section 2.3.1., using phasor notation instead of time dependent functions. The circulating currents obtained for each type of transposition considered are presented in Table 3-9.

Table 3-9 - Circulating currents at rated speed.

<table>
<thead>
<tr>
<th>Type of transposition</th>
<th>Strand number</th>
<th>Circulating current [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC – 2 strands per row</td>
<td>1</td>
<td>1.025e^{58.3°}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.566e^{123°}</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.415e^{-157.2°}</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.075e^{-140.4°}</td>
</tr>
<tr>
<td>CTC – 3 strands per row</td>
<td>1</td>
<td>0.405e^{122.6°}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.587e^{12.6°}</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.002e^{-167.4°}</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.130e^{-159.3°}</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.015e^{30.2°}</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.769e^{-173.9°}</td>
</tr>
<tr>
<td>CTC – 4 strands per row</td>
<td>1</td>
<td>1.242e^{39.9°}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.340e^{33.0°}</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.853e^{23.5°}</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.267e^{23.9°}</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.143e^{151.5°}</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.317e^{144.9°}</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>1.594e^{145.5°}</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.673e^{-165.1°}</td>
</tr>
<tr>
<td>CTC – 5 strands per row</td>
<td>1</td>
<td>0.683e^{32.2°}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.203e^{195.2°}</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.080e^{-19.4°}</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.632e^{26.0°}</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.418e^{-158.0°}</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.921e^{-157.8°}</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.134e^{22.9°}</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.229e^{24.7°}</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.269e^{-161.0°}</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.051e^{140.8°}</td>
</tr>
<tr>
<td>Roebel – 2 strands per row</td>
<td>1</td>
<td>$1.411e^{-142.4}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.012e^{-140.0}$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$4.267e^{-29.3}$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$1.917e^{-162.5}$</td>
</tr>
<tr>
<td>Roebel – 3 strands per row</td>
<td>1</td>
<td>$7.407e^{-154.4}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$3.753e^{-20.7}$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$1.202e^{-147.8}$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$1.246e^{-119.3}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$0.852e^{-75.4}$</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$5.517e^{-26.9}$</td>
</tr>
<tr>
<td>Roebel – 4 strands per row</td>
<td>1</td>
<td>$2.538e^{-22.8}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.607e^{-23.1}$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$0.549e^{-13.3}$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$0.340e^{-141.8}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$3.149e^{-157.9}$</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$0.430e^{-170.0}$</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$0.754e^{-143.8}$</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>$0.222e^{127.4}$</td>
</tr>
<tr>
<td>Roebel – 5 strands per row</td>
<td>1</td>
<td>$0.656e^{-156.0}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.346e^{-160.1}$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$0.830e^{-28.8}$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$1.886e^{-1157.0}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$1.405e^{-25.6}$</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$1.294e^{-1156.5}$</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$1.398e^{23.9}$</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>$0.889e^{-147.7}$</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>$1.856e^{21.7}$</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>$0.586e^{7.5}$</td>
</tr>
</tbody>
</table>
3.2.3. Losses results

The resistive losses is finally given by equation (2.17) and calculated individually for each strand, including the circulating current, before adding it up to find the losses of the half turn. Total resistive losses of the machine is simply given by equation (3.1). Results are presented in Table 3-10.

\[ P_{j}^{\text{Total}} = 2P_{j}^{\text{half-turn}} \cdot N_{c} = 2 \left( \sum P_{j}^{\text{strand}} \right) \cdot N_{c} \cdot (mp) \]  

(3.1)

Table 3-10 - Total resistive losses.

<table>
<thead>
<tr>
<th>Transposition type</th>
<th>Total losses [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC – 2 strands per row</td>
<td>498.4</td>
</tr>
<tr>
<td>CTC – 3 strands per row</td>
<td>443.4</td>
</tr>
<tr>
<td>CTC – 4 strands per row</td>
<td>418.3</td>
</tr>
<tr>
<td>CTC – 5 strands per row</td>
<td>399.0</td>
</tr>
<tr>
<td>Roebel – 2 strands per row</td>
<td>665.7</td>
</tr>
<tr>
<td>Roebel – 3 strands per row</td>
<td>570.0</td>
</tr>
<tr>
<td>Roebel – 4 strands per row</td>
<td>501.0</td>
</tr>
<tr>
<td>Roebel – 5 strands per row</td>
<td>467.7</td>
</tr>
<tr>
<td>Litz wire</td>
<td>341.8</td>
</tr>
</tbody>
</table>

3.3. Rotational losses

Considering a straight conductive filament exposed to a magnetic field whose variation is the same along both filament’s length and thickness, then the eddy losses in such conductor can be calculated directly by equation (2.47) along the whole volume. However, in an ironless stator AFPM machine, the flux density waveform is not a pure sinusoid. This fact, along with the air gap magnetic field geometry exhibiting a 3-D nature (i.e. different behavior along machine’s axial and radial directions), implies that calculating eddy current losses merely analytically is subjected to significant errors [10]. Moreover, in a transposed winding, field orientation taken for the calculation varies according to strand positioning. Taking as reference the subconductor axial section, the two magnetic field components of interest for (2.47) must be parallel to strand’s section height and width, which are not the same as the axial and tangential components seen from the machine perspective. For instance, magnetic field coordinates in a strand shifting from one row to the other are different from the ones in a strand shifting upwards. Consequently, the analysis of each subconductor has to be performed discretizing its volume in such a manner that the above particularities are accounted for. Using basic trigonometry on strand’s segment crossed by the slice under analysis, it is possible to find magnetic field coordinates according to strand’s section. Such calculations are explained in section 3.4.3 of this report.

Figure 3-10 illustrates the approach chosen for volume discretization. To account for the flux density waveform in the air gap being close to a trapezoid, which varies along the axial direction of the machine, the concept of layer is introduced. To account for the flux density varying along the radii of the machine, the concept of slice is introduced. Each slice is an arc along the pole pitch and is built from several layers, which basically are copies of each other displaced along strand’s cross section thickness.
In the active region, group of turns from 1 to N/2 and from N/2+1 to N experience the same flux density variation. Consequently, losses in upper and lower turn groups can be inferred solely from turns 1 and N, respectively. This is not valid in the end winding region, since the perimeter of the arc connecting half turns is larger in outer turns (e.g. turns 1 and N) than in inner turns (e.g. turns N/2 and N/2+1). Although this points to studying each strand of each turn individually, this would increase computational efforts unworthily, since field variation in end region is negligible when compared to the active zone. Likewise, differences between turns in there may be ignored and the whole analysis reduced to turns 1 and N, referred as lower and upper turn, respectively.

The procedure followed to calculate eddy rotational losses is described below:

i. discretize strand length into slices;
ii. discretize strand height into layers;
iii. extract axial and tangential magnetic field along each layer;
iv. compute an average layer representative of the segment of the conductor in play;
v. for the average layer, perform Fourier analysis along pole-pitch for axial and tangential magnetic field to obtain \( \bar{B}_x \) and \( \bar{B}_y \) of equation (2.47);
vi. integrate equation (2.47) over the dimensions of the conductor in play to obtain the losses within one slice;
vii. losses within the strand (half turn) is the losses of all slices:

\[
\Delta P_{\text{strand}} = \sum_{k=1}^{i} \Delta P_{\text{slice}_k}
\]  

(3.2)
where $s$ is the total number of slices;

viii. losses within the half turn is the sum of the losses in each of its strands:

$$\Delta P_{\text{turn}} = \sum_{k=1}^{st} \Delta P_{\text{strands}_k}$$

(3.3)

where $st$ is the total number of strands;

ix. losses within the phase coil is the losses of every turn:

$$\Delta P_{\text{phase}} = 2 \times \left[ \frac{N}{2} \Delta P_{\text{coint}} + \frac{N}{2} \Delta P_{\text{coint}} \right]$$

(3.4)

x. total losses is the sum of losses in every coil:

$$\Delta P_{\text{total}} = nc \times \Delta P_{\text{phase}}$$

(3.5)

where $nc$ is the total number of coils, given by the product between the number of phases and the number of pole pairs.

Due to the rotary symmetry, only one pole-pitch is necessary to model the entire machine [10].

Figure 3-11 and Figure 3-12 illustrate the volume distribution of magnetic induction vector in the machine section.

![Figure 3-11](image)

Figure 3-11 - Profile and front views of the magnetic induction vector.
Figure 3-12 - Volume distribution of magnetic induction vector.

Figure 3-13 and Figure 3-14 illustrate the magnetic induction vector and magnitude, respectively, along selected layers of the machine. Figure 3-15 shows the magnitude of X, Y and Z global axis components of the magnetic induction vector.
Figure 3-13 - Magnetic induction vector along selected layers of the machine.

Figure 3-14 - Magnetic induction magnitude along selected layers of the machine.
Figure 3-15 - From left to right: magnitude of X, Y and Z global axis components of the magnetic induction vector.

Figure 3-16 and Figure 3-17 show the change on those components when the geometrical correction to calculate field components accordingly to winding positioning is applied. Layer numeration was kept accordingly to the software which means the lower the identification number is, the smaller the radius of the layer (e.g. closer to the origin of the global axis).

Figure 3-16 - field components of layer 2.
As it can be seen by the change in X and Z components of the field after the adjustment, layer 10 corresponds to a segment of the winding that is leaning along the thickness direction.

### 3.3.1. Losses results

Below, the total numerical results obtained when applying the algorithm described above to the computational model are presented in Figure 3-18 and Figure 3-19. Partial losses per strand can be found in Appendix C.
Figure 3-18 - Total rotational losses of the machine.

Figure 3-19 - Total rotational losses of the machine at rated speed.
3.4. Algorithms

Some important details of the code used to perform calculations over the three-dimensional models are discussed. The bits of code referring to the two-dimensional models are disregarded in this description as they are straightforward enough to be understood with the concepts presented for the three-dimensional models.

3.4.1. Central lines of transposed strands

Figure 3-22 shows the fluxogram of function TranspCTC, which takes last points form the central line of a conductor and computes the next transposition step by updating the cell array containing central lines of each strand with the corresponding new line segments that result from transposition. The necessary transposition step is simply evaluated by the strand current positioning in the winding, which results in a simple algorithm. However, there is one complex detail worthy of explanation. As shown in Figure 3-20, length of the central line of a strand depends on its position on the winding, being $a$ the thickness of a strand, $\delta$ the length difference of two adjacent strands in the same row and $dzx$ the displacement in the winding's length direction when a strand shifts position within the same row. Thus, when extending the strand before bending it in a vertical shift, there must be a parameter to weigh such displacement. This parameter is referred to as “prmtr” in the fluxogram below. Naturally, strands with a lower extension before shifting up or down will have a larger extension after the bent, to complete the transposition step. This is a very important aspect in the building of the three-dimensional model of the winding because central lines are the paths along which cross sections are swept to form the volume of the subconductors, and if this difference in length is not taken into account, strands intersect each other, making it impossible for the model to successfully pass software pre-simulation validation criteria.

In order to determine $\delta$, two ways of calculating the area of the blue triangle in Figure 3-21 are considered. Such calculation is shown in set of equations (3.6).
\[
\text{Area}_a = \frac{bh}{2} \\
\text{Area}_{\text{case1}}^a = \frac{\left(\sqrt{dz^2 + a^2}\right)}{2} a \\
\text{Area}_{\text{case2}}^a = \frac{(dz + ?)}{2} a \\
\text{Area}_{\text{case1}}^a = \text{Area}_{\text{case2}}^a \Rightarrow ? = \sqrt{dz^2 + a^2} - dz
\] (3.6)

To compute the parameter referred in Figure 3-22 it should be noted that, within a row in which vertical shifts are going up, the lower a strand is located the larger the length of the central line is before the bend. Similarly, in a row in which vertical shifts are going down, the higher a strand is located the larger the length of the central line is before the bend. Furthermore, the decreases in length of the strands are multiple of each other. For instance, in a row shifting up, ? of the third lowest strand is twice as much as the ? of the second lowest strand.

Figure 3-21 - Determination of the length difference of two adjacent strands in the same row.
Figure 3-22 - Fluxogram of algorithm to perform one transposition step of a CTC winding.

Figure 3-23 shows the fluxogram of function TranspR, which takes initial points form the central line of a conductor and computes all the transposition steps by updating the cell array containing central lines of each strand with the corresponding new line segments that result from transposition. The nature of the task is simple, as the complete transposition path is solely determined by the strand initial positioning in the winding, which results in a straightforward algorithm.
Figure 3-23 - Fluxogram of algorithm to perform a complete transposition of a Roebel winding.
3.4.2. Arc paths forming the layers for eddy losses calculation

Figure 3-24 shows the fluxogram of function EddyGeom, which takes points that form the central line of a conductor and computes three points from the arc of the respective layer used in rotational losses calculation. Two of these three points are located in the ends of the segment under analysis, while the third one is in the middle. None of the points forming the central line of the conductor is in the referred positions as the stranded winding is located in the quarter of the segment. Consequently, calculations have to be performed in order to determine the arc paths. Actually, the point of the arc located in the middle of the segment could be replaced by the original point from the subconductor as the arc also contains the latter. However, it is useful to shift the winding to a central position to determine the layers of segments of the central line corresponding to shifts within and between rows. The central line of each subconductor is analyzed by segments of two points in order to ensure that at least one slice is positioned at each transposition step. This is important because, as already seen, the rotor field varies significantly with the spatial orientation of the conductor.

To rotate a point with coordinates \( p_x \) and \( p_y \) along the Z axis until the Y coordinate origin is reached (i.e. \( p_y^{new} = 0 \)), system of equations (3.7) must be solved, where the unknowns are the new X coordinate \( p_x^{new} \) and the rotation angle \( ang \).

\[
\begin{align*}
  p_x^{new} &= p_x \cos(ang) - p_y \sin(ang) \\
  p_y^{new} &= 0 = p_x \sin(ang) + p_y \cos(ang)
\end{align*}
\]

Once the segment is rotated, the second step is to determine where in the segment the slices are going to be. Firstly, the direction vector of the segment must be calculated as it allows determining any point within the segment it is directing. For instance, given points A and B from opposite ends of a line segment, with coordinates \( a_x, a_y, a_z \) and \( b_x, b_y, b_z \), respectively, the direction vector of the segment is given by equation (3.8). Coordinates \( c_x, c_y, c_z \) of any point C belonging to the segment are given by set of equations (3.9), where \( m \) is the equation slope. For \( m = 0 \), A is obtained, while B is found with \( m = 1 \).

\[
dv = [b_x - a_x; b_y - a_y; b_z - a_z]
\]

\[
\begin{align*}
  c_x &= a_x + m dv_x \\
  c_y &= a_y + m dv_y \\
  c_z &= a_z + m dv_z
\end{align*}
\]
Figure 3-24 - Fluxogram of algorithm followed to build layers used in eddy losses calculation.
Following the same logic, positioning of the first slice (i.e. the one nearest to the first point of the segment) can be determined by solving system of equations (3.10), with the equation slope $m$ also as an unknown and where \( ds \) is the slice discretization step. After knowing the coordinates of the first slice and the equation slope, the computation of the coordinates of the remaining slices is straightforward.

\[
\begin{align*}
  p_{x_{\text{slice}}} &= p_{x} + m \cdot dv_{x} \\
  p_{z_{\text{slice}}} &= p_{z} + m \cdot dv_{z}
\end{align*}
\]

\[
\sqrt{(p_{x} - p_{x_{\text{slice}}})^2 + (p_{z} - p_{z_{\text{slice}}})^2} = ds
\]

Although slightly more complex, the computation of the coordinates of the layers follows the same principles described for the slices. It should be noted that layer's central point position is firstly determined as belonging to the segment and only then is rotated to its true out of the segment place. First layer is computed using the exact same approach as in system of equations (3.10), only replacing \( ds \) by the layer discretization step \( dl \). Then the procedure is repeated, replacing \( ds \) by half of strand's thickness, in order to find the maximum height for the layer. Using both equations slopes that result from the solution of the two previous systems, it is possible to determine layers positioning within the subconductor. Finally, it is necessary to rotate the found coordinates +90° and -90°. Coordinates are given accordingly to the global coordinate system. However, in order to perform rotations that will take layers to their true positions the reference for rotation must be the axis of the slice. Consequently, the procedure to correctly achieve rotations is firstly to perform a translation by taking the point near the origin of the global coordinate system (i.e. as far from it as it originally is from the slice); then, perform the rotation following the method described by set of equations (3.7); and finally perform the inverse translation in order to take the point back to its original position near the subconductor.

### 3.4.3. Rotational losses calculation

Figure 3-25 shows the fluxogram of function RotLosses, which takes parameters specifying the type of winding and the dimensions of the machine to compute the total eddy losses, as well as the losses within each strand on both lower and upper turns. Active region central lines are drawn using function TranspCTC or TranspR, depending on the transposition type, while the link between active and end region is achieved as follows. The available data for this purpose regards the model with a non-transposed winding, made of a massive cross section, thus a relation had to be found between the line linking both regions in that situation and the position of central lines of each strand in the transposed coil. Set of equations (3.11) shows the coordinates of the six points from the central line of a massive cross section winding that form the end winding region. As illustrated in Figure 3-26, points 1, 2 and 3 form the inner end region, while points 4, 5 and 6 correspond to the outer end region. Naturally, points 1 and 4 are also part of the active region. It should be noted that the coordinates of these points are given already considering the true position of the coil, which is rotated along the Z axis by \( \theta \), i.e. a quarter of the angle of one segment of the machine containing one pole pair \( \theta \).
Figure 3-25 - Fluxogram of algorithm followed to calculate eddy losses.
\[ p^1 = (p^1_x; p^1_y; p^1_z) = \left( \frac{D_i}{2} \cos(\theta_w); -\frac{D_i}{2} \sin(\theta_w); 0 \right) \]
\[ p^2 = (p^2_x; p^2_y; p^2_z) = \left( \frac{D_i}{2} - 2w \right) \cos(\theta_w); -p^2_z \tan(\theta_w); h \]
\[ p^3 = (p^3_x; p^3_y; p^3_z) = \left( \frac{D_i}{2}, p^3_z + \frac{\pi(D_i - 2w)}{p}; p^3_z \right) \]
\[ p^4 = (p^4_x; p^4_y; p^4_z) = \left( \frac{D_o}{2} \cos(\theta_w); -\frac{D_o}{2} \sin(\theta_w); 0 \right) \]
\[ p^5 = (p^5_x; p^5_y; p^5_z) = \left( \frac{D_o}{2} + 2w \right) \cos(\theta_w); -p^5_z \tan(\theta_w); h \]
\[ p^6 = (p^6_x; p^6_y; p^6_z) = \left( p^6_x, p^6_z + \frac{\pi(D_o + 2w)}{p}; p^6_z \right) \]

\( D_i \) and \( D_o \) are the inner and outer diameter of the machine, respectively. \( w \) is the winding width, \( h \) is the winding thickness and \( p \) the number of poles of the machine.

Figure 3-26 - Points from the central line of a massive cross section winding that form the end winding region.

To compute both end regions for the central line of each strand, the direction vector \( dv \) of the central line from the non-stranded conductor must be calculated and then a parallel line can be found for each strand. The length of these lines depends on the distance from the center of the strand to the center of the winding, and it is this parameter that works as the equation slope \( m \). As illustrated in Figure 3-27, the further a strand is from the center of the coil, the larger its end region central line is. Set of equations (3.12) exemplifies how to find points
1, 2 and 3 from the inner end winding region for each strand. Naturally, the procedure for the outer region is similar.

\[
    dv = [p_2^2 - p_1^1, p_3^2 - p_1^1, p_2^2 - p_1^1]
\]

\[
    \text{length} = \sqrt{(p_3^2 - p_1^1)^2 + (p_2^2 - p_1^1)^2 + (p_2^2 - p_1^1)^2}
\]

\[
    \text{dist}_{\text{central}}^{\text{strand}} = p_1^1 - p_1^1
\]

\[
    m = \frac{\text{length} + \text{dist}_{\text{central}}^{\text{strand}}}{\text{length}}
\]

\[
    p_2^{\text{str}} = (p_2^{\text{str}1}; p_2^{\text{str}2}; p_2^{\text{str}3}) = (p_2^{\text{str}1} + m.dv(1); p_2^{\text{str}1} + m.dv(2); p_2^{\text{str}1} + m.dv(3))
\]

\[
    p_3^{\text{str}} = (p_3^{\text{str}1}; p_3^{\text{str}2}; p_3^{\text{str}3}) = (p_2^{\text{str}1}; p_2^{\text{str}2}; p_2^{\text{str}3} + \|p_2^2 - p_2^a\| + \frac{\pi(D_i - 2w)}{p}; p_2^{\text{str}3})
\]

Figure 3-27 - Difference in length of end region central line of each strand.

The width and thickness of one strand are \( L = w_{\text{str}}/2 \) and \( A = h_{\text{str}}/n_{\text{str}} \), respectively, where \( w_{\text{str}} = w_f(N/2) \) is the width of one turn, \( h_{\text{str}} = h/2 \) is the thickness of one turn and \( n_{\text{str}} \) is the number of strands per row. As the filling factor \( k_f \) is 0.5, surface determined for every strand has to be equally divided by two layers: one inner conductive made of copper and a surrounding layer of nonconductive material (e.g. vacuum). This is illustrated in Figure 3-28. System of equations (3.13) shows how to calculate the new width \( l \) and thickness \( a \) of the conductive surface. \( l_c \) and \( a_c \) are also unknowns.
PMs are built by sweeping a profile of the cross section along an arc path. Set of equations (3.14) show coordinates of the three points that form the central line arc of a PM, which is located at the central radius of the machine, i.e. \( r_m = \frac{(D_1/2 + D_2/2)}{2} \). The magnet pitch factor \( k_m \) indicates the fraction of the angle of a pole occupied by a PM, i.e. \( \theta_m = \frac{2\pi}{p} k_m \). To draw the arc as the central line of the PM, the distance from the surface of the winding to the surface of the PM, \( g \), and the magnet thickness \( h_m \) must also be taken into account. It should be noted that the coordinates below regard the magnets in the lower side of the double-sided rotor (i.e. side with negative Z coordinate). Also, the central line is not at its true position. Consequently, after being created by sweeping a profile over the arc path, the PM must be rotated around Z axis by an angle of

\[
\frac{2\pi}{p} \left(1 - k_m \right) + \frac{\theta_m}{2}.
\]
\[ p^1 = (p_1^1; p_1^2; p_1^3) = (r_{av} \cos \left( -\frac{\theta_M}{2} \right); r_{av} \sin \left( -\frac{\theta_M}{2} \right); -\left( g + \frac{h + h_M}{2} \right) ) \]

\[ p^2 = (p_2^1; p_2^2; p_2^3) = (r_{av}; 0; p_1^1) \]

\[ p^3 = (p_3^1; p_3^2; p_3^3) = (r_{av} \cos \left( \frac{\theta_M}{2} \right); r_{av} \sin \left( \frac{\theta_M}{2} \right); p_1^1) \]

(3.14)

Naturally, the center of this arc is not the axis origin, but \((x; y; z) = (0; 0; -\left( g + \frac{h + h_M}{2} \right) )\). Finally, the arc profile has an area of \( h_M \times (D_a - D_t / 2) \). Analogously, the three points that form the arc necessary to build the rotor are given by set of equations (3.15), where \( h_{Fe} \) is the iron thickness.

\[ p^1 = (p_1^1; p_1^2; p_1^3) = (r_{av} \cos \left( -\frac{\theta}{2} \right); r_{av} \sin \left( -\frac{\theta}{2} \right); -\left( g + \frac{h + h_M + h_{Fe}}{2} \right) ) \]

\[ p^2 = (p_2^1; p_2^2; p_2^3) = (r_{av}; 0; p_1^1) \]

\[ p^3 = (p_3^1; p_3^2; p_3^3) = (r_{av} \cos \left( \frac{\theta}{2} \right); r_{av} \sin \left( \frac{\theta}{2} \right); p_1^1) \]

(3.15)

Magnetic field components stored in Maxwell's Fields Calculator are given according to global axis coordinates. What is necessary for the losses calculation purpose is those components but according to the strand's cross section position. Consequently, an adjustment to change the orientation of those components must be performed. Only then may the harmonics be computed and used in the eddy losses equation. To achieve this purpose the schematic of one sector of the machine represented in Figure 3-29 is considered.

![Figure 3-29 - Geometry considered for the coordinate transformations of the magnetic induction components.](image-url)
Figure 3-30 illustrates the coordinate transformation that takes the global X and Y components and turns each of them into field components oriented along the winding length and width. Set of equations (3.16) accomplish such transformation.

\[
\theta^x_{G-arc} = \frac{L_{arc}}{R_{slice}} \\
B_{Wlength} = B_x \cos\left(\theta^x_{G-arc}\right) \\
B_{Width} = B_x \sin\left(\theta^x_{G-arc}\right) \\
\theta^y_{G-arc} = \pi/2 - \theta^x_{G-arc} \\
B_{Wlength} = B_{Wlength} + B_y \cos\left(\theta^y_{G-arc}\right) \\
B_{Width} = B_{Width} + B_y \sin\left(\theta^y_{G-arc}\right)
\] (3.16)

Is should be noted that the array returned by the software with the induction components along the layer is given according to a distance measured from one side of the segment to the other. That is not the measure of the arc length of interest to make the coordinates adjustment. Thus, when \(C_{dist} \leq R_{slice} \theta/2\), then \(L_{arc} = R_{slice} \theta/2 - C_{dist}\) and when \(C_{dist} > R_{slice} \theta/2\), then \(L_{arc} = C_{dist} - R_{slice} \theta/2\).

After the previously presented transformation, it is necessary to check if the slice under study corresponds to a vertical or horizontal shift, and act accordingly. Set of equations (3.17) shows the procedure in case of a shift in
the same row, while shifts between different rows are covered by set of equations (3.18). Both these coordinate transformations are illustrated in Figure 3-31 and Figure 3-32, respectively.

\[
\theta_{\text{Vshift}}^{\text{Wlength}} = \arcsin \left( \frac{\sqrt{(p_{z}^{n+1} - p_{z}^{n})^2}}{\sqrt{(p_{x}^{n+1} - p_{x}^{n})^2 + (p_{y}^{n+1} - p_{y}^{n})^2 + (p_{z}^{n+1} - p_{z}^{n})^2}} \right)
\]

\[
B_{\text{Vshift}}^{\text{Wlength}} = B_{\text{Vshift}}^{\text{Wlength}} \cos(\theta_{\text{Vshift}}^{\text{Wlength}})
\]

\[
B_{\text{Vshift}}^{\text{Vshift}} = B_{\text{Vshift}}^{\text{Vshift}} \sin(\theta_{\text{Vshift}}^{\text{Vshift}})
\]

\[
\theta_{\text{Vshift}}^{\text{Width}} = \pi/2 - \theta_{\text{Vshift}}^{\text{Wlength}}
\]

\[
B_{\text{Vshift}}^{\text{Vshift}} = B_{\text{Vshift}}^{\text{Vshift}} + B_{\text{Vshift}}^{\text{Width}} \cos(\theta_{\text{Vshift}}^{\text{Width}})
\]

\[
B_{\text{Vshift}}^{\text{Width}} = B_{\text{Vshift}}^{\text{Width}} + B_{\text{Vshift}}^{\text{Width}} \sin(\theta_{\text{Vshift}}^{\text{Width}})
\]

Figure 3-31 - Coordinate transformation that takes the field component oriented along the winding length and global Z and adjusts each of them to the actual length and thickness of the segment of winding subjected to a vertical shift. The green and yellow angles correspond to \(\theta_{\text{Vshift}}^{\text{Wlength}}\) and \(\theta_{\text{Vshift}}^{\text{Width}}\), respectively.

\[
\theta_{\text{Hshift}}^{\text{Wlength}} = \arcsin \left( \frac{\sqrt{(p_{y}^{n+1} - p_{y}^{n})^2}}{\sqrt{(p_{x}^{n+1} - p_{x}^{n})^2 + (p_{y}^{n+1} - p_{y}^{n})^2 + (p_{z}^{n+1} - p_{z}^{n})^2}} \right)
\]

\[
B_{\text{Hshift}}^{\text{Wlength}} = B_{\text{Hshift}}^{\text{Wlength}} \cos(\theta_{\text{Hshift}}^{\text{Wlength}})
\]

\[
B_{\text{Hshift}}^{\text{Width}} = B_{\text{Hshift}}^{\text{Width}} \sin(\theta_{\text{Hshift}}^{\text{Width}})
\]

\[
\theta_{\text{Hshift}}^{\text{Width}} = \pi/2 - \theta_{\text{Hshift}}^{\text{Wlength}}
\]

\[
B_{\text{Hshift}}^{\text{Wlength}} = B_{\text{Hshift}}^{\text{Wlength}} + B_{\text{Hshift}}^{\text{Width}} \cos(\theta_{\text{Hshift}}^{\text{Width}})
\]

\[
B_{\text{Hshift}}^{\text{Width}} = B_{\text{Hshift}}^{\text{Width}} + B_{\text{Hshift}}^{\text{Width}} \sin(\theta_{\text{Hshift}}^{\text{Width}})
\]
3.4.4. Resistive losses calculation

Figure 3-33 shows the fluxogram of function ResLosses, which takes parameters specifying the type of winding and its dimensions to compute the impedance of each strand for an array of frequencies under analysis. Central lines of each strand are drawn using function TranspCTC or TranspR, depending on the transposition type. The three-dimensional conductors are obtained by sweeping cross sectional profiles along the path formed by central lines. While for CTC transposition type it is only necessary one profile at one of the ends of the winding in order to achieve a clean sweep operation, for a Roebel winding several sweeps have to be performed along the segments of the central line. This is due to the obliques geometry of the winding that prevents a complete sweep operation to be performed along the total length of the strand without intersections between different objects.
Figure 3-33 - Fluxogram of algorithm followed to compute the impedance of each strand.
3.4.5. Flux linkage determination for circulating currents calculation

Figure 3-34 shows the fluxogram of function cc, which takes machine dimensions and computes a transient model in order to determine the flux linked with each subconductor. Procedures to design the rotor and the active region half turn are exactly as described for functions RotLosses and ResLosses, respectively. Additionally, the algorithm defines model objects required for every time-varying design. The idea then is to rotate the double-sided rotor at rated speed during the time needed for it to travel along one segment. It is possible to achieve acceptable results analyzing one pole pair segment only by using the symmetry multiplier capabilities, which allows the simulation to run within an efficient time.

Figure 3-34 - Fluxogram of algorithm followed to compute flux linkage.
Chapter 4

Conclusions

This chapter is aimed at detailing overall conclusions of the thesis as well as presenting future work perspectives.
4.1. Main ideas

The purpose of this work was to assess the feasibility of using two conventional and cheaper alternatives to Litz wire in the stator winding of a 10 MW AFPM machine. Obtained results are quite clear and, as already expected before the study, the conclusion is that both proposed technologies are still far from delivering the losses reduction performance provided by Litz wire. The reason behind this is the considerable difference in the cross section surface available for the current to flow. As in Roebel winding and CTC additional space is needed to make the transposition physically possible, there is a waste of the available surface, which leads to an increase in the winding resistance and a subsequent rise in the resistive losses. Naturally, this waste is larger the lower the number of strands is, since surface of the strands is bigger. As Roebel transposition requires one more strand of additional space than CTC, resistive losses have the pace presented in Figure 4-1, where a direct relation between this kind of losses and the available conductive surface is easily noted. On the other hand, the much smaller cross section surface of subconductors in Litz wire implies this technology is using almost ten times more strands than the best solutions of the other two options, reducing the resistive losses to an unreachable competitive edge. A curious and, at first, unexpected result of this work is the inverse proportionality between

![Figure 4-1 - Total winding losses balance.](image)
the number of strands and obtained rotational losses, as illustrated in Figure 4-1. It would seem logic to think that if subdividing the conductor in several subconductors serves the purpose of reducing eddy current effects, then the more intense that division is, the sharpest rotational losses reduction would be. It is true that using a stranded conductor instead of a massive one provides better rotational losses performance, as it will be shown further in this section. However, since a stranded wire is built from several parallel circuits, transposition has to be performed in order that the induced EMFs in all parallel conductors are equalized, consequently reducing circulating currents. This fact implies that the available conductive surface is larger in the cases with more sub conductors, thus increasing rotational losses. Another fact that may contribute to this increase with the number of strands is that, as explained in section 3.4.3 of this report, when the conductor is transposed, the changes in its dimensions orientation along the transposition path may cause the components of the flux that contribute to the rotational losses to mutual increase each other. Understandably, components of the flux may also align in a way they cancel each other out. The number of winding direction changes is proportional to the number of strands, and what these results indicate is that, in this two specific types of transposition, most direction changes in transposition steps cause the flux components to increase their effective value for the rotational losses. Figure 4-1 indicates that, at the end, an apparent random pace is followed by total winding losses, which implies a balance must be made between resistive and rotational losses in order to find the best solution for given dimensions. Nevertheless, it is not clear that the process of subdividing winding conductors is work that pays off in terms of total winding losses reduction. Table 4-1 shows the gain in transposing the winding weighting it against the case with a massive cross section, which dimensions are 973.5 mm of length and 19.15 mm² of available conductive surface. Recalling its resistance, already presented in section 3.2.1, a total resistive losses of 339.9 kW is found. Same approach described in section 3.3 of this report was followed for the no stranded conductor and 42.9 kW was obtained for the rotational losses.

### Table 4-1 - Comparison between transpositions weighted against no transposed winding.

<table>
<thead>
<tr>
<th>Winding type</th>
<th>Surface [%]</th>
<th>Length [%]</th>
<th>Resistive losses [p.u.]</th>
<th>Rotational losses [p.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Massive cross section</td>
<td>100</td>
<td>100</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CTC – 4 strands</td>
<td>66.7</td>
<td>100.23</td>
<td>1.46</td>
<td>0.19</td>
</tr>
<tr>
<td>CTC – 6 strands</td>
<td>75</td>
<td>100.3</td>
<td>1.30</td>
<td>0.22</td>
</tr>
<tr>
<td>CTC – 8 strands</td>
<td>80</td>
<td>100.34</td>
<td>1.23</td>
<td>0.27</td>
</tr>
<tr>
<td>CTC – 10 strands</td>
<td>83.3</td>
<td>100.37</td>
<td>1.17</td>
<td>0.33</td>
</tr>
<tr>
<td>Roebel – 4 strands</td>
<td>50</td>
<td>100.03</td>
<td>1.96</td>
<td>0.15</td>
</tr>
<tr>
<td>Roebel – 6 strands</td>
<td>60</td>
<td>100.03</td>
<td>1.68</td>
<td>0.21</td>
</tr>
<tr>
<td>Roebel – 8 strands</td>
<td>66.7</td>
<td>100.03</td>
<td>1.47</td>
<td>0.27</td>
</tr>
<tr>
<td>Roebel – 10 strands</td>
<td>71.4</td>
<td>100.04</td>
<td>1.38</td>
<td>0.34</td>
</tr>
</tbody>
</table>

An issue that should be discussed is the advantages of using a tridimensional simulation. It should be pondered if it is possible or not to achieve similar results with a two-dimensional model and if the computational effort spent is worthwhile. Table D-1 of Appendix D gives a general idea of the computational effort required to run each of the model analysis used in this report, in terms of used memory and completion time. Rotational losses calculation in AFPM machines is usually described in the literature (e.g. [10] and [15]) as a two-dimensional problem. However, following the same method in a three-dimensional environment provides more accurate results not only due to the flux density varying along the radii of the machine, but also because it is not possible...
to account for the magnetic flux variation within a transposed coil using a two-dimensional model. In terms of resistive losses, there is now no doubt similar results could be achieved merely by recurring to the analytical expression that gives the resistance of a conductor. This happens because of the low frequencies under study. Basically, skin, proximity and spirality effects are unnoticeable and the impedance of the winding is simply only a function of its dimensions. However, the extremely heavy three-dimensional simulation run was not in vain since self and mutual inductances of the subconductors were needed for circulating currents determination. Highly dependent on the geometry, this parameter requires a three-dimensional model. Even considering that including circulating currents in the resistive losses calculation is not a crucial step, as shown in Table 4-2, this is not a reason to underrate the method because it can still be used to determine how big transposition angle has to bring the losses to a desired level.

Table 4-2 - Weight of circulating currents in Joule losses.

<table>
<thead>
<tr>
<th>Transposition type</th>
<th>Circulating currents contribution [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC – 4 strands</td>
<td>0.17</td>
</tr>
<tr>
<td>CTC – 6 strands</td>
<td>0.21</td>
</tr>
<tr>
<td>CTC – 8 strands</td>
<td>0.78</td>
</tr>
<tr>
<td>CTC – 10 strands</td>
<td>0.11</td>
</tr>
<tr>
<td>Roebel – 4 strands</td>
<td>0.52</td>
</tr>
<tr>
<td>Roebel – 6 strands</td>
<td>3.17</td>
</tr>
<tr>
<td>Roebel – 8 strands</td>
<td>0.85</td>
</tr>
<tr>
<td>Roebel – 10 strands</td>
<td>0.87</td>
</tr>
</tbody>
</table>

4.2. What remains to be done

An analysis of circulating currents in a stranded but no transposed winding is clearly missing in this report to assess about how worth 360 degrees transposition is. Furthermore, and although the type of generator under consideration is a low frequency machine, it would be interesting to have performed calculations under frequencies that make skin, proximity and spirality effects measurable. This would not only allow concluding about the usefulness of Roebel winding and CTC for other applications, but would also provide proof or rebuttal about the general consideration that by stranding a tubular conductor the skin effect is reduced as the number of strands is increased, providing the number of subconductors is greater than seven [16]. But even with that, the work developed in this study would be far from being complete. In fact, it only represents a small first step to the final ambition of creating an algorithm that outputs the cheapest solution for winding technology given a maximum value of accepted losses. It is safe to say that this study serves as solid background to accomplish the referred big final task because principles that support these algorithms were built over a parameterized basis. In other words, the exact same scripts may be used to perform the same kind of calculations for other machines with different dimensions. Several future works may follow, for instance to study different types of transposition that makes use of the fact that the field varies over the radii of the machine, or integrate an economic analysis with winding losses calculation to evaluate the maximum degree before deciding which type of winding will be part of the stator.
The resistance limited eddy losses in the stator of an AFPM machine may be experimentally determined by measuring the difference in input shaft powers of the AFPM machine at the same speed, first with the stator in, and then by replacing it with a dummy stator (no conductors). The dummy stator has the same dimensions and surface finish as the real stator and is meant to keep the windage losses the same. The shaft of the tested prototype and the shaft of the driving machine are coupled together via a torque meter. The stator is positioned in the middle of the two rotor discs with the outer end ring mounted on the outside supporting frame. Initially, the prototype with the coreless stator placed in it is driven by a variable speed motor for a number of different speeds and the corresponding torque measurements are taken. Replacing the real stator with a dummy one, the tests are repeated for the same speeds. Using the logic expressed in equation (2.22), the difference of the torques multiplied by the speed gives the eddy current losses. The eddy losses due to eddy-circulating currents in the windings can also be determined by measuring the difference of input shaft powers, first with all the parallel circuits connected and then with disconnected parallel circuits [4]. For a number of reasons related with the priority on the access to the prototype of the machine, it was not possible to complete the experimental test by the time this report was submitted. However, a wooden stator, whose scheme is illustrated in Appendix E, was already built and practical results are expected to be included in a future paper. At the moment, the next steps are upgrading the shaft line, setting the torque sensor, and replacing the real stator.
References


[16]. HB. Dwight, Skin Effect and Proximity Effect in Tubular Conductors, Transactions AIEE, 1922.
Appendix A

Formulas from vector analysis
A.1 Gradient

The gradient vector field of a scalar function \( f(x_1, \ldots, x_n) \) is defined as the unique vector field whose dot product with any unit vector at each point is the directional derivative of the function along that unit vector. In a rectangular coordinate system, the gradient is the vector field whose components are the partial derivatives of \( f \):

\[
\nabla f = \frac{\partial f}{\partial x_1} e_1 + \ldots + \frac{\partial f}{\partial x_n} e_n
\]

\( e_i \) are the orthogonal unit vectors pointing in the coordinate directions.

In terms of physical interpretation, the variation in space of any quantity can be represented by a slope and the gradient represents the steepness and direction of that slope.

A.2 Divergence

Being \( x, y, z \) a system of Cartesian coordinates in 3-dimensional Euclidean space, and \( \vec{i}, \vec{j}, \vec{k} \) the corresponding basis of unit vectors, then the divergence of a continuously differentiable vector field \( \vec{F} = U\vec{i} + V\vec{j} + W\vec{k} \) is equal to the scalar-valued function:

\[
\nabla \cdot \vec{F} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}
\]

In terms of physical interpretation, the divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.

A.3 Curl

Expanded in Cartesian coordinates, the curl of a continuously differentiable vector field \( \vec{F} \) composed of \( [F_x, F_y, F_z] \) is equal to the vector field:

\[
\nabla \times \vec{F} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
F_x & F_y & F_z
\end{vmatrix}
\]

\( \vec{i}, \vec{j}, \vec{k} \) are the unit vectors for the \( x, y, z \) axes, respectively. This expands as follows:
\[ \nabla \times \vec{F} = \left( \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \hat{i} + \left( \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \hat{j} + \left( \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{k} \]

In terms of physical interpretation, at every point in the field \( \vec{F} \), the curl is represented by a vector. The length and direction of this vector characterize the rotation at that point. The direction of the curl is the axis of rotation, as determined by the right-hand rule, and the magnitude of the curl is the magnitude of rotation.

**A.4 Gauss’ theorem**

The Gauss’ theorem relates the volume integral of the divergence of a vector field \( \vec{D} \) over a volume \( V \) to the surface integral over a closed surface \( S_v \), bounding a region of volume \( V \), of the dot product of \( \vec{D} \) and the outward unit normal \( \vec{n}_v \).

\[ \int_V \nabla \cdot \vec{D} \, dV = \int_{S_v} \vec{D} \cdot \vec{n}_v \, dS \]

**A.5 Stokes’ theorem**

The Stokes’ theorem relates the surface integral of the curl of a vector field \( \vec{A} \) over a surface \( S \) in Euclidean three-space to the line integral of the vector field over its boundary \( \vec{s} \):

\[ \int_{S} \nabla \times \vec{A} \cdot \vec{n}_s \, dS = \oint_{s} \vec{A} \cdot d\vec{s} \]
Appendix B

Resistive losses: resistance
CTC – 6 strands

Figure B-1 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed CTC with 3 strands per row.

Table B-1 - Resistance [mOhm] of each strand in a non-transposed CTC winding with 3 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 3, 4, 6</td>
<td>7.013121</td>
<td>7.013129</td>
<td>7.013144</td>
<td>7.013215</td>
<td>7.013497</td>
<td>7.013968</td>
<td></td>
</tr>
<tr>
<td>2, 5</td>
<td>7.013121</td>
<td>7.013133</td>
<td>7.013154</td>
<td>7.013254</td>
<td>7.013653</td>
<td>7.014318</td>
<td></td>
</tr>
</tbody>
</table>

Figure B-2 - Distribution of the magnitude of current density vector over a CTC with 3 strands per row.

Table B-2 - Resistance [mOhm] of each strand in a CTC winding with 3 strands per row.

<table>
<thead>
<tr>
<th>Strand</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.034400</td>
<td>7.034402</td>
<td>7.034406</td>
<td>7.034411</td>
<td>7.034421</td>
<td>7.034482</td>
<td>7.034584</td>
</tr>
<tr>
<td>2</td>
<td>7.034424</td>
<td>7.034425</td>
<td>7.034429</td>
<td>7.034434</td>
<td>7.034444</td>
<td>7.034506</td>
<td>7.034609</td>
</tr>
<tr>
<td>3</td>
<td>7.034455</td>
<td>7.034457</td>
<td>7.034461</td>
<td>7.034466</td>
<td>7.034476</td>
<td>7.034536</td>
<td>7.034637</td>
</tr>
<tr>
<td>4</td>
<td>7.034309</td>
<td>7.034311</td>
<td>7.034314</td>
<td>7.034319</td>
<td>7.034329</td>
<td>7.034391</td>
<td>7.034493</td>
</tr>
<tr>
<td>5</td>
<td>7.034330</td>
<td>7.034332</td>
<td>7.034336</td>
<td>7.034341</td>
<td>7.034351</td>
<td>7.034413</td>
<td>7.034515</td>
</tr>
<tr>
<td>6</td>
<td>7.034415</td>
<td>7.034417</td>
<td>7.034420</td>
<td>7.034426</td>
<td>7.034436</td>
<td>7.034497</td>
<td>7.034598</td>
</tr>
</tbody>
</table>
CTC – 8 strands

Figure B-3 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed CTC with 4 strands per row.

Table B-3 - Resistance [mOhm] of each strand in a non-transposed CTC winding with 4 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DC 15 25 36 50 100 150</td>
</tr>
<tr>
<td>1, 4, 5, 8</td>
<td>8.766401 8.766407 8.766419 8.766438 8.766472 8.766683 8.767036</td>
</tr>
<tr>
<td>2, 3, 6, 7</td>
<td>8.766401 8.766410 8.766426 8.766453 8.766500 8.766798 8.767294</td>
</tr>
</tbody>
</table>

Figure B-4 - Distribution of the magnitude of current density vector over a CTC with 4 strands per row.
Table B-4 - Resistance [mOhm] of each strand in a CTC winding with 4 strands per row.

<table>
<thead>
<tr>
<th>Strand</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.796564</td>
<td>8.796567</td>
<td>8.796572</td>
<td>8.796581</td>
<td>8.796635</td>
<td>8.796725</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.796592</td>
<td>8.796596</td>
<td>8.796601</td>
<td>8.796610</td>
<td>8.796664</td>
<td>8.796756</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8.796559</td>
<td>8.796564</td>
<td>8.796569</td>
<td>8.796577</td>
<td>8.796631</td>
<td>8.796721</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8.796509</td>
<td>8.796514</td>
<td>8.796519</td>
<td>8.796527</td>
<td>8.796580</td>
<td>8.796669</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>8.796649</td>
<td>8.796654</td>
<td>8.796658</td>
<td>8.796667</td>
<td>8.796720</td>
<td>8.796808</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8.796650</td>
<td>8.796655</td>
<td>8.796660</td>
<td>8.796668</td>
<td>8.796721</td>
<td>8.796810</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8.796612</td>
<td>8.796617</td>
<td>8.796622</td>
<td>8.796630</td>
<td>8.796684</td>
<td>8.796773</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8.796642</td>
<td>8.796647</td>
<td>8.796652</td>
<td>8.796660</td>
<td>8.796713</td>
<td>8.796801</td>
<td></td>
</tr>
</tbody>
</table>

CTC – 10 strands

Figure B-5 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed CTC with 5 strands per row.

Table B-5 - Resistance [mOhm] of each strand in a non-transposed CTC winding with 5 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 5, 6, 10</td>
<td>10.519681</td>
<td>10.519686</td>
<td>10.519695</td>
<td>10.519711</td>
<td>10.519738</td>
<td>10.519907</td>
<td>10.520190</td>
</tr>
</tbody>
</table>
Figure B-6 - Distribution of the magnitude of current density vector over a CTC with 5 strands per row.

Table B-6 - Resistance [mOhm] of each strand in a CTC winding with 5 strands per row.

<table>
<thead>
<tr>
<th>Strand</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>10.559329</td>
<td>10.559330</td>
<td>10.559332</td>
<td>10.559336</td>
<td>10.559342</td>
<td>10.559382</td>
<td>10.559449</td>
</tr>
</tbody>
</table>
Roebel – 4 strands

Figure B-7 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed Roebel with 2 strands per row.

Table B-7 - Resistance [mOhm] of each strand in a non-transposed Roebel winding with 2 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3, 4</td>
<td>DC</td>
</tr>
<tr>
<td>1, 2, 3, 4</td>
<td>7.013121</td>
</tr>
</tbody>
</table>

Figure B-8 - Distribution of the magnitude of current density vector over a Roebel with 2 strands per row.

Table B-8 - Resistance [mOhm] of each strand in a Roebel winding with 2 strands per row.

<table>
<thead>
<tr>
<th>Strand</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DC</td>
</tr>
<tr>
<td>1</td>
<td>7.018260</td>
</tr>
<tr>
<td>2</td>
<td>7.018263</td>
</tr>
<tr>
<td>3</td>
<td>7.018264</td>
</tr>
<tr>
<td>4</td>
<td>7.018263</td>
</tr>
</tbody>
</table>
Roebel – 6 strands

Figure B-9 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed Roebel with 3 strands per row.

Table B-9 - Resistance [mOhm] of each strand in a non-transposed Roebel winding with 3 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 3, 4, 6</td>
<td>8.766401</td>
<td>8.766407</td>
<td>8.766418</td>
<td>8.766435</td>
<td>8.766467</td>
<td>8.766666</td>
<td>8.766996</td>
</tr>
<tr>
<td>2, 5</td>
<td>8.766401</td>
<td>8.766409</td>
<td>8.766424</td>
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<td>8.766491</td>
<td>8.766760</td>
<td>8.767208</td>
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</tbody>
</table>

Figure B-10 - Distribution of the magnitude of current density vector over a Roebel with 3 strands per row.
Table B-10 - Resistance [mOhm] of each strand in a Roebel winding with 3 strands per row.

<table>
<thead>
<tr>
<th>Strand</th>
<th>DC</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>8.773547</td>
<td>8.773550</td>
</tr>
<tr>
<td>2</td>
<td>8.773549</td>
<td>8.773552</td>
</tr>
<tr>
<td>3</td>
<td>8.773549</td>
<td>8.773552</td>
</tr>
<tr>
<td>4</td>
<td>8.773545</td>
<td>8.773548</td>
</tr>
<tr>
<td>5</td>
<td>8.773547</td>
<td>8.773550</td>
</tr>
<tr>
<td>6</td>
<td>8.773545</td>
<td>8.773548</td>
</tr>
</tbody>
</table>

Roebel – 8 strands

![Current Density Distribution](image)

Figure B-11 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed Roebel with 4 strands per row.

Table B-11 - Resistance [mOhm] of each strand in a non-transposed Roebel winding with 4 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DC</td>
</tr>
<tr>
<td>1, 4, 5, 8</td>
<td>10.519681</td>
</tr>
<tr>
<td>2, 3, 6, 7</td>
<td>10.519681</td>
</tr>
</tbody>
</table>

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Figure B-12 - Distribution of the magnitude of current density vector over a Roebel with 4 strands per row.

Table B-12 - Resistance [mOhm] of each strand in a Roebel winding with 4 strands per row.

<table>
<thead>
<tr>
<th>Strand</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
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<tr>
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<td>10.528875</td>
<td>10.528878</td>
<td>10.528883</td>
<td>10.528916</td>
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</tbody>
</table>

Roebel – 10 strands

Figure B-13 - Distribution of the magnitude of current density vector at 36 Hz over the cross sections of a nontransposed Roebel with 5 strands per row.
Table B-13 - Resistance [mOhm] of each strand in a non-transposed Roebel winding with 5 strands per row.

<table>
<thead>
<tr>
<th>Strands</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 5, 6, 10</td>
<td>12.272961</td>
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<td>12.272973</td>
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Figure B-14 - Distribution of the magnitude of current density vector over a Roebel with 5 strands per row.

Table B-14 - Resistance [mOhm] of each strand in a Roebel winding with 5 strands per row.

<table>
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<th>Strand</th>
<th>DC</th>
<th>15</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
</table>
Appendix C

Rotational losses per strand
CTC – 4 strands

Figure C-1 - Rotational losses of a CTC with 4 strands.

CTC – 6 strands

Figure C-2 - Rotational losses of a CTC with 6 strands.
CTC – 8 strands

Figure C-3 - Rotational losses of a CTC with 8 strands.

CTC – 10 strands

Figure C-4 - Rotational losses of a CTC with 10 strands.
Roebel – 4 strands

Figure C-5 - Rotational losses of a Roebel winding with 4 strands.

Roebel – 6 strands

Figure C-6 - Rotational losses of a Roebel winding with 6 strands.
Roebel – 8 strands

Figure C-7 - Rotational losses of a Roebel winding with 8 strands.

Roebel – 10 strands

Figure C-8 - Rotational losses of a Roebel winding with 10 strands.
Appendix D

Computational effort
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<th>Model</th>
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<th>Completion time [hh:mm:ss]</th>
<th>Solution type</th>
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Appendix E

Dummy stator
Figure E-1 - Scheme of the wooden dummy stator.